

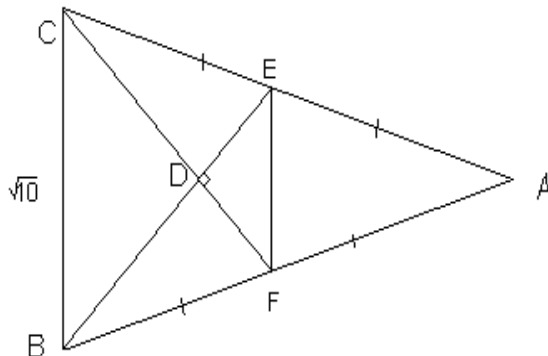
Math Field Day 2002

2002 HUDDLE – Solutions

1. Eleven years passed between the time when my father's age and mine summed to my present age, and the time when my father's age and mine summed to his present age. How old was my father when I was born?

Solution: Whenever 11 years passes, the sum of my father's age and mine increases by 22 years. Therefore my father is 22 years older than I.

2. Isosceles triangle ABC has $AB = AC$, base $BC = \sqrt{10}$, and its medians from vertices B and C are perpendicular. Find the length of side AC . (A median in a triangle is the line through a vertex and the midpoint of the opposite side.)



Solution: By symmetry, BDC is an isosceles right triangle, so $CD = BD = \sqrt{5}$. Triangles BDC and FDE are similar and $EF = BC/2$, so $DE = \sqrt{5}/2$. Therefore $CE^2 = 5 + 5/4 = 25/4$, so $AC = 2CE = 5$.

3. What is the minimum value of the function $f(x) = |x+5| + |x-3| + |x-7|$?

Solution: Note that $|x+5| + |x-7|$ is the total distance from x to -5 and to 7 , which is 12 if x is between -5 and 7 , and greater otherwise. Therefore the minimum value of f is taken on at $x = 3$, where $f(x) = 12$.

4. Given that $3m + 2n = mn$, where m and n are positive integers, list all possible values of m .

Solution: Solving for n , we obtain $n = \frac{3m}{m-2} = 3 + \frac{6}{m-2}$, so $m-2$ must be $1, 2, 3$ or 6 , so $m = 3, 4, 5$ or 8 . (The solutions (m, n) are $(3, 9)$, $(4, 6)$, $(5, 5)$, $(8, 4)$.)