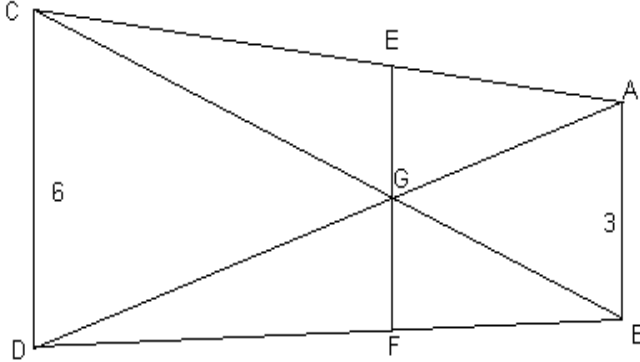


## Leapfrog 2002

1. In the figure below, lines  $AB$ ,  $CD$  and  $EF$  are all parallel. If  $AB$  has length 3, and  $CD$  has length 6, find the length of  $EF$ .



2. Suppose that  $f(x) = (100!)^{g(x)}$ , where

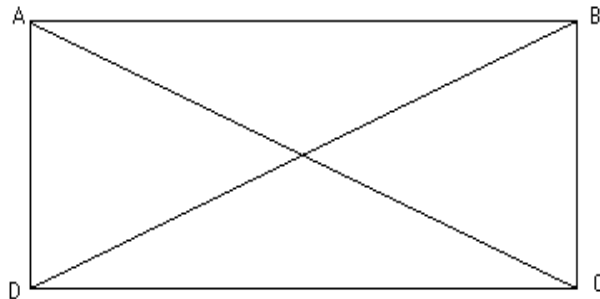
$$g(x) = \frac{100}{\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x} + \frac{1}{\log_5 x} + \cdots + \frac{1}{\log_{100} x}}.$$

Express  $f(x)$  in the simplest possible form.

3. Evaluate this ratio of infinite sums as a fraction in lowest terms:

$$\frac{1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \frac{1}{6^3} + \cdots}{1 - \frac{1}{2^3} + \frac{1}{3^3} - \frac{1}{4^3} + \frac{1}{5^3} - \frac{1}{6^3} + \cdots}.$$

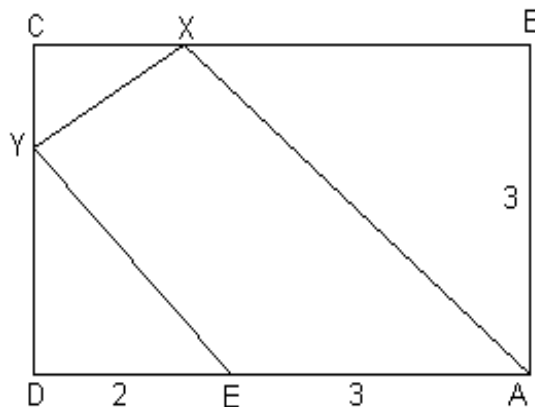
4. Each of the six line segments  $AB$ ,  $AC$ ,  $AD$ ,  $BC$ ,  $BD$  and  $CD$  in the figure below is to be colored either red or green. How many ways are there to do this so that each of the four triangles  $ABC$ ,  $ABD$ ,  $ACD$  and  $BCD$  has at least one red side and one green side?



5. If  $a_1 = a_2 = 0$ , simplify the expression

$$\sum_{k=1}^{\ell} \sum_{j=1}^k (a_{j+2} - 2a_{j+1} + a_j).$$

6. In rectangle  $ABCD$ , the length of  $AB$  is 3, and the length of  $BC$  is 5. Sides  $BC$  and  $CD$  are mirrors. You want to shine a beam of light from vertex  $A$  so that it reflects off of sides  $BC$  at point  $X$ , reflects off of side  $CD$  at point  $Y$ , and finally hits side  $AD$  at point  $E$ , which is 3 units from  $A$ . How far is  $X$  from  $C$ ? (Hint: The laws of reflection require that  $\angle AXB = \angle YXC$  and  $\angle XYC = \angle EYD$ .)



7. What is the remainder when  $x^{100}$  is divided by  $(x - 1)^2$ ?

8. On a multiple choice test, one of the questions was illegible. The choice of answers was:

- (a) All of the below.
- (b) None of the below.
- (c) All of the above.
- (d) Exactly one of the above.
- (e) None of the above.
- (f) None of the above.

List all of the correct answers.