

## Huddle 2006 solutions

1. Suppose the grid is filled in so that every row, column, and  $2 \times 2$  box in double lines contains each of the digits 1,2,3 and 4. Evaluate  $X$ .

	1		
2			
3		$X$	
			4

Solution: The correct diagram is

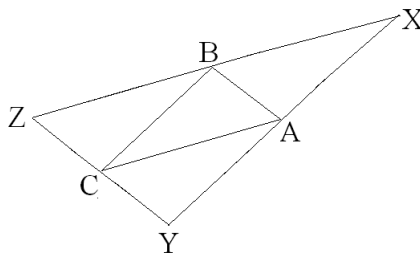
4	1	2	3
2	3	4	1
3	4	1	2
1	2	3	4

$X = 1$ .

2. The number  $30X0X03$ , written in ordinary decimal notation, is a multiple of 13. What digit is  $X$ ?

Solution:  $3000003 = 13 \cdot 230769 + 6$ , and  $10100 = 13 \cdot 776 + 12$ , so  $30X0X03 = 13(230769 + 776X) + 12X + 6$ .  $12X + 6$  must be a multiple of 13, which is true for  $X = 6$ .

3. Triangle  $ABC$  has area 1. Form a new triangle  $XYZ$  by constructing on each vertex of triangle  $ABC$  a line parallel to the opposite side. What is the area of triangle  $XYZ$ ?



Solution: The parallel lines guarantee that each small triangle has corresponding angles equal to those of triangle  $ABC$ . Each small triangle also shares a side with  $ABC$ , and so is congruent to  $ABC$ . The area of triangle  $XYZ$  is 4.

4. Die  $A$  is 6-sided, and has the number 1 printed on 5 of its faces, and the number 2 printed on the remaining face. Die  $B$  is 4-sided, and has the number  $X$  printed on 3 of its faces, and  $Y$  on the remaining face. On both dice, each face is equally likely to come up.

When you roll the two dice, the probability is  $1/3$  that the sum of the numbers that come up is 5. Evaluate  $X$ .

Solution: When you roll the two dice, there are  $6 \cdot 4 = 24$  possible outcomes. 15 of them sum to  $1 + X$ , 3 to  $2 + X$ , 5 to  $1 + Y$ , and 1 to  $2 + Y$ . We need 8 outcomes to sum to 5; this requires that  $2 + X = 1 + Y = 5$ , so  $X = 3, Y = 4$ .