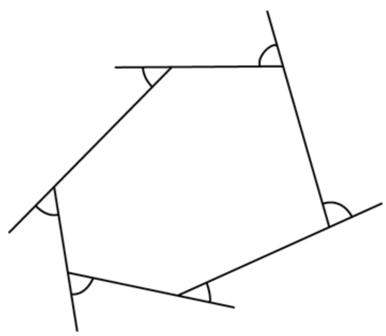
2015 MATH FIELD DAY
LEAPFROG PROBLEMS

Problem 1. Let i denote a square root of -1, and consider the sequence of numbers defined by $y_0 = 1$, $y_1 = 5$, and $y_n = iy_{n-1} + y_{n-2}$ for $n \geq 2$. Evaluate $\dfrac{y_{2015}}{y_{2009}}$.

Problem 2. In a polygon, the exterior angles are the angles supplementary to the interior angles, as in the figure below.



In a given regular n-gon, each interior angle measures exactly 7 times that of each exterior angle. Find n.

Problem 3. Let $r(x)$ be the remainder after diving x^{2015} by $x^2 - x$. Find $r(2)$.

Problem 4. An arithmetic sequence of positive integers contains the numbers 6 and 122, though 122 is not the term directly after 6. There are no perfect squares in the sequence between the terms 6 and 122. What is the next number in the sequence after 122?

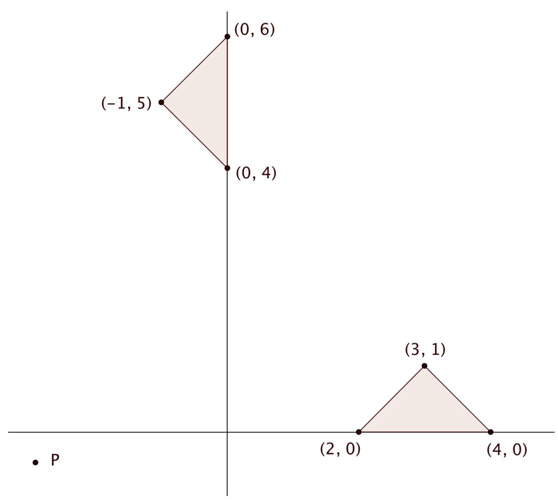
Problem 5. Find the smallest positive real solution x (in radians) to the equality

$$\sqrt{3}\cos(x) - \sin(x) = \sqrt{2}.$$

Problem 6. Find the largest integer $m < 45$ such that m^m divides evenly into 45^{45}.

Problem 7. Find the radius of a circle inscribed in a triangle with two sides of length 17 and one side of length 16.

Problem 8. A rotation of the plane around a point P takes the triangle with vertices $(2, 0)$, $(4, 0)$, and $(3, 1)$ to the triangle with vertices $(0, 4)$, $(0, 6)$, and $(-1, 5)$. Find the coordinates of P.



Note: Not drawn to scale/accuracy.