Math Field Day 2015
Mad Hatter A

A1.

$$4^{x+2} = 8^{-x+1}$$

$$x = ?$$

A2. Alistair was half as old as Bertha was when Bertha was 2 years older than Alistair will be in 5 years. Their present ages sum to 46. How old is Bertha?

A3. 10% of people are left-handed. Two people are selected at random from the U.S. population. What is the probability that at least one of them is left-handed?

A4. Suppose that for all x, $f(x) = 5x + 2$ and $f(g(x)) = x$. If $g(y) = f(y)$, evaluate y.

A5. Simplify completely $5^{\log_3(7^{\log_5 3})}$.

A6. The circle $x^2 + y^2 = 2$ has tangent line $y = -x + k$, where k is a positive number. $k = ?$

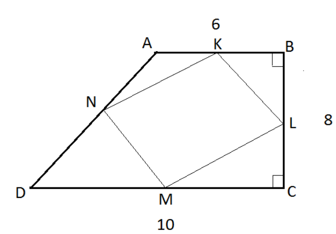
A7.

$$x^{10}y^3 = 12$$

$$5\log_2 x + \log_2 y = 1$$

$$y = ?$$

A8.



Quadrilateral $ABCD$ has right angles at B and C. $AB = 6$, BC=8, $CD = 10$. Points K, L, M, N are the midpoints of its sides. Find the area of quadrilateral $KLMN$.

A9. Evaluate

$$\frac{1+2+4+8+\cdots+512}{1+4+16+64+256}.$$

A10.

$$\tan\theta\sec\theta = 3/8$$
$$\sin\theta = ?$$

A11.
Find the largest positive integer n such that

$$(n+2)! \leq 10,000 \cdot n!$$

A12.

$$\log_2 x + \log_4 x = 9$$
$$x = ?$$

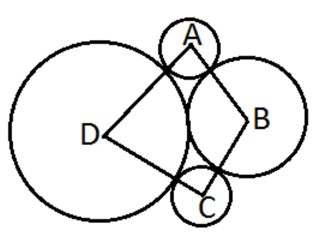
A13. Find the diameter of the circle through the points $(0,0)$, $(4,0)$ and $(0,3)$.

A14.

$$\frac{x-y}{x+y} = 2. \qquad \frac{x-2y}{x+2y} = ?$$

A15. a and b are integers. $\sqrt{a+12\sqrt{7}} = 3+b\sqrt{7}$. $a = ?$

A16.



The pictured circles with centers at A, B, C, D have radii 1,2,1,3, respectively. Find the area of quadrilateral $ABCD$.

A17. The greater root of

$$x^2 + bx + 128 = 0$$

is twice the other. $b =?$

A18. Suppose that $x > 1$ and $y > 1$. Simplify

$$\log_{xy}\left((x+y)^2\right) - 2\log_{xy}(x^{-1} + y^{-1}).$$

A19. $k > 0$.
The triangle with vertices

$$(0,0), \quad (1,-1), \quad (k,k)$$

has area 6.

$$k =?$$

A20.
The lines $y = 2x - 3$ and $y = 0$ form angle θ.

$$\sin^2\theta =?$$

A21. $\{a_n\}$ is an arithmetic sequence.

$$a_1 + a_2 + a_3 = 3$$

$$a_1 + a_2 + a_3 + a_4 = 10$$

$$a_1 + a_2 + a_3 + a_4 + a_5 =?$$

A22. d and e are integers.

$x - \sqrt{2}$ is a factor of $x^4 + 3x^3 - 5x^2 + dx + e$.

$$d =?$$

A23. Find the probablility that flipping 4 fair coins gives 2 heads and 2 tails.

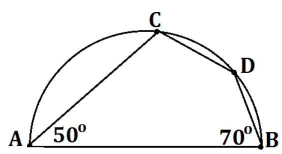
A24.

$$\frac{\log x + \log y}{\log 5 + \log 20} = 1$$

$$\frac{x}{y} = \frac{9}{100}$$

$$x = ?$$

A25.



C and D lie on a semicircle with endpoints A and B, as shown. Evaluate the ratio of line segment lengths CD/AB.

A26. s and t are the roots of $x^2 - 24x + 51 = 0$.
$s - 2$ and $t - 2$ are the roots of $x^2 + ax + b = 0$.

$$b = ?$$

A27.
The circle $x^2 + kx + y^2 - ky = 7$ has radius 5. If $k > 0$, then $k = ?$

A28.
How many real numbers satisfy the equation $|x - 2| \;=\; |\,|x - 5| - 1\,|$?

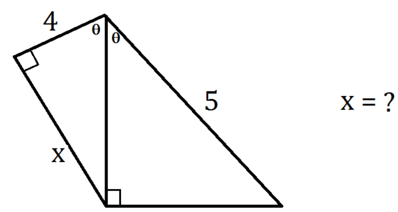
A29.
The number

$$\frac{\log_5 4 - \log_5 2}{\log_5 16 - \log_5 4 + \log_5 8}$$

is rational. Express it as a fraction in lowest terms.

A30.
Evaluate $\sin^4(2015^o) - \cos^4(2015^o) - 2\sin^2(2015^o)$.

A31.



A32. b and c are real numbers, and x^2+2x+c is a factor of x^3+5x^2+b. $b=?$

A33.
Find the area of region bounded by the graphs of $x=6-|y|$ and $x=2+|y|$.

A34.
Find the sum of all roots of $x^4-x^2-2=0$.

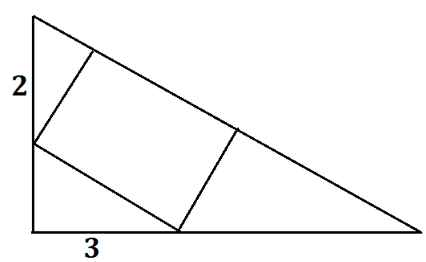
A35.
What is the minimum x value of any point on the parabola

$$(y+4)^2+2=4(x+1) \quad ?$$

A36.

$$2^{1/\log_5 2}+5^{1/\log_2 5}= \ ?$$

A37.



A rectangle inscribed in a right triangle creates segments of lengths 2 and 3 on the legs as shown. Find the area of the rectangle.

A38. For how many integers n, $1\le n\le 100$, does the graph of $f(x)=(x-2)^{n-1}(x-4)$ touch but not cross the x-axis at the point $(2,0)$?

A39.

What is the domain of the function $f(\theta) = \ln\left(\left(\dfrac{\sin\theta}{2}\right)^2 - \sin^2\left(\dfrac{\theta}{2}\right)\right)$?

A40.

$$204^2 - 199^2 = \ ?$$

Math Field Day 2015

Mad Hatter B

B1.

$$4^{x+2} = 8^{-x+1}$$

$$x = ?$$

B2. Alistair was half as old as Bertha was when Bertha was 2 years older than Alistair will be in 5 years. Their present ages sum to 46. How old is Bertha?

B3. 10% of people are left-handed. Two people are selected at random from the U.S. population. What is the probability that at least one of them is left-handed?

B4. Suppose that for all x, $f(x) = 5x + 2$ and $f(g(x)) = x$. If $g(y) = f(y)$, evaluate y.

.

B5.

$$\frac{2019}{2018 - \frac{2017}{2015-79x}} = \frac{2019}{2018 - \frac{2017}{2016-78x}}$$

$$x = ?$$

B6. The circle $x^2 + y^2 = 2$ has tangent line $y = -x + k$, where k is a positive number. $k = ?$

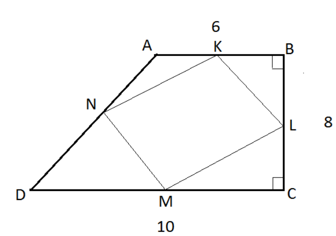
.

B7. x and y are integers.

$$x^2 - y^2 = 7$$

$$x^2 + y^2 = ?$$

B8.



Quadrilateral $ABCD$ has right angles at B and C. $AB = 6$, BC=8, $CD = 10$. Points K, L, M, N are the midpoints of its sides. Find the area of quadrilateral $KLMN$.

B9. Evaluate

$$\frac{1+2+4+8+\cdots+512}{1+4+16+64+256}.$$

.

B10.
Simplify completely

$$[(x^{-1}-1)^{-1}+1]^{-1}.$$

B11.
Find the largest positive integer n such that

$$(n+2)! \leq 10,000 \cdot n!$$

.

B12.

$$\begin{array}{ccccccccc} w & + & x & + & y & & & = & 3 \\ & & x & + & y & + & z & = & 2 \\ w & & & + & y & + & z & = & 1 \\ w & + & x & & & + & z & = & 0 \end{array}$$

$$x = ?$$

B13. Find the diameter of the circle through the points $(0,0)$, $(4,0)$ and $(0,3)$.

B14.

$$\frac{x-y}{x+y} = 2. \qquad \frac{x-2y}{x+2y} = ?$$

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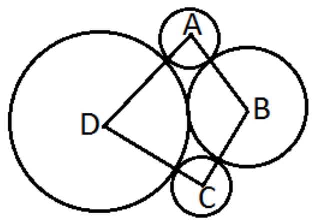

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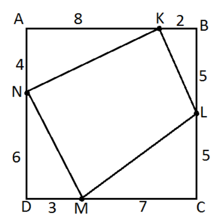

$ABCD$ is a 10×10 square. Points $KLMN$ lie on its sides, and divide the sides as shown. What is the area of quadrilateral $KLMN$?

B19. $k > 0$.
The triangle with vertices

$$(0,0), \quad (1,-1), \quad (k,k)$$

has area 6.

$$k = ?$$

B20.
The lines $y = 2x - 3$ and $y = 0$ form angle θ.

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B22. d and e are integers.

$x - \sqrt{2}$ is a factor of $x^4 + 3x^3 - 5x^2 + dx + e$.

$$d = ?$$

B23. Find the probablility that flipping 4 fair coins gives 2 heads and 2 tails.

.

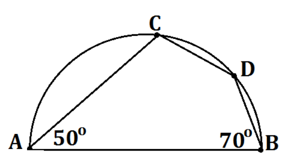
B24. What is the greatest possible number of times the graphs of the polynomial functions

$$f(x) = a_{2015}x^{2015} + a_{2014}x^{2014} + \cdots + a_0$$

$$\text{and} \quad g(x) = b_2x^2 + b_1x + b_0$$

can intersect, if $a_{2015} \neq 0$?

B25.



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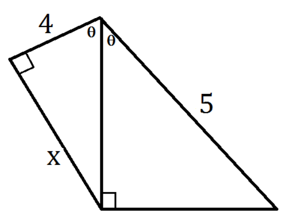
The circle $x^2 + kx + y^2 - ky = 7$ has radius 5. If $k > 0$, then $k = ?$

B28.
How many real numbers satisfy the equation $|x-2| \ = \ |\ |x-5|-1\ |$?

B29.
$x^2 + bx + c$ has $3 + i$ as a root, where $i^2 = -1$.
b and c are both real numbers.
Find b.

B30.
Find the smallest positive integer n such that $n^2 \geq 11n + 70$.

B31.



x = ?

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Find the area of region bounded by the graphs of $x = 6-|y|$ and $x = 2+|y|$.

B34.
Find the sum of all roots of $x^4 - x^2 - 2 = 0$.

B35.
What is the minimum x value of any point on the parabola

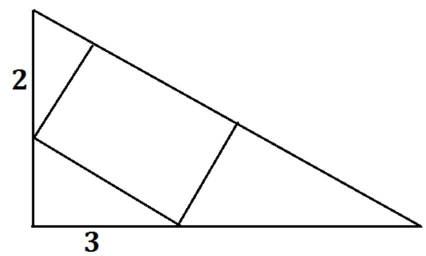
$$(y+4)^2 + 2 = 4(x+1) \ \ ?$$

B36. For all real numbers x,

$$f(x) + 2f(1-x) = x.$$

$$f(-1) =?$$

B37.



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B38. For how many integers n, $1 \leq n \leq 100$, does the graph of $f(x) = (x-2)^{n-1}(x-4)$ touch but not cross the x-axis at the point $(2, 0)$?

B39. How many integers n satisfy the inequalities

$$|n| < |n-8| \quad \text{and} \quad |n| < |n+6|?$$

B40.

$$204^2 - 199^2 = \ ?$$