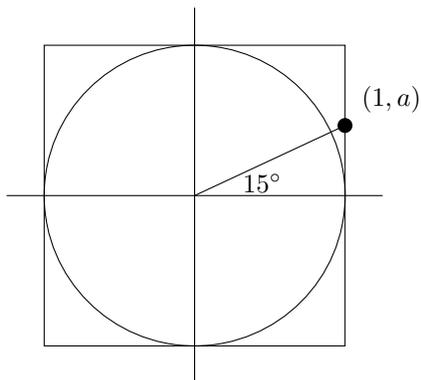**2017 MATH FIELD DAY**
LEAP FROG PROBLEMS

Problem 1. A rectangle has the following properties:

- Its length and width are positive integers, neither of which is an integer multiple of the other.
- Its area is three times its perimeter.

Find its area.

Problem 2. The figure below shows the unit circle and a point marked on the circumscribed unit square.

What integer is $\left(\frac{1}{a+1} + \frac{1}{a-1}\right)^{-2}$?

Problem 3. For any real number x, let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. For example, we have $\lfloor 2.19 \rfloor = 2$ and $\lfloor 3 \rfloor = 3$. In effect, $\lfloor x \rfloor$ rounds x down to an integer.
Consider the sequence defined by $a_1 = 2017$ and for $n > 1$,

$$a_n = 1 + n \left\lfloor \frac{a_{n-1} - 1}{n} \right\rfloor.$$

Find a_{2017}.

Problem 4. Consider the polynomial

$$(3x + 5)^6(x^2 + x - 2)^4(4 + 3x)^8 = a_{22}x^{22} + a_{21}x^{21} + a_{20}x^{20} + \cdots + a_2x^2 + a_1x + a_0.$$

of degree 22. Find $a_0 + a_2 + a_4 + \cdots + a_{22}$.

Problem 5. Suppose m and n are positive integer solutions to the equation
$$mn + 3n + 5m - 16 = 2017.$$
How many possible values for m are there?

Problem 6. Consider a two-player game that begins with a large number of coins in a pile. On each player's turn, they can remove anywhere from 1 to 2017 coins from the stack. Whoever takes the last coin wins.

If the game begins with 2,020,200 coins in a pile, the first player can guarantee eventual victory with perfect play, but only by playing the correct first move. How many coins should the first player take on the first turn?

Problem 7. Begin with a solid cube and carve out (remove) a square pyramid whose base is one face of the cube and whose vertex is the center of the opposite face. The resulting solid has smaller volume but larger surface area then the original cube. If the volume decreased by the same amount that the surface area increased, what is the side length of the cube?
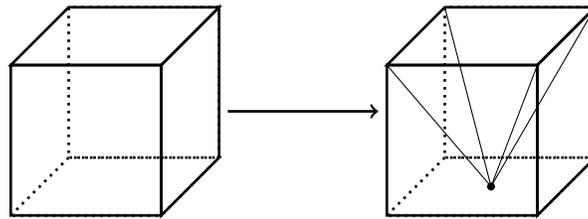
Problem 8. On a trip to the mythical Fields of Mathematics, Artur comes across the infamous three-headed Pemdas of Doom. Each of its three heads, named Manjul, Martin, and Maryam, speak only one sentence per day, and whether they speak the truth or lie is determined solely by the day of the week. Manjul tells the truth only on Monday through Thursday. Martin tells the truth only on Tuesdays, Thursdays, and Saturdays. Maryam tells the truth only on the weekend.

The beast's three heads address Artur as follows:

> **Manjul**: I will tell the truth 700 days from now.

> **Martin**: Manjul lied 100 days ago, and will lie 100 days from now as well.

> **Maryam**: I will lie 1000 days from now.

What day of the week is it?