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Warmup 1.
If $x = 1 + \dfrac{1}{1 + \frac{1}{1+1}}$,

evaluate $\dfrac{1}{\frac{1}{x-1} - 1}$.

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Warmup 2.
There are 4 fnorks in a grelb.
There are 11 veeblefesters in a fnork.
How many veeblefester-grelbs are there in a fnork^2?

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Warmup 3.
You have a collection of 100 red chips and 100 blue chips. You randomly split them into a small pile of 50 chips and a large pile of 150 chips. How many more red chips are there in the large pile than blue chips in the small pile?

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Warmup 4.
It is a fact that $101^2 = 10201$.
Evaluate 10101^2.

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B1.

Factor completely:

$$[\,(x+2)(x+3)(x+4)\,]^2 - [\,(x+1)(x+2)(x+3)\,]^2$$

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B2. The fraction $\frac{1}{3}$ has the property that when you add its denominator to both its numerator and its denominator you get $\frac{4}{6}$, which is exactly twice the original fraction. What fraction (in lowest terms) has the property that when you add its denominator to both its numerator and its denominator, the fraction that you get is exactly three times the original fraction?

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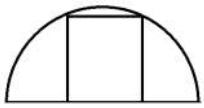
B3.



Let A be the area of a semicircle and B the area of the square inscribed in the semicircle. Find the value of $\dfrac{A}{B}$.

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B4.
I am 3 years older than my brother will be next year, when I will be twice his present age. How old am I?

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B5. There are four candidates for the mayor of a city, A, B, C, D. Suppose A is twice as likely to be elected as B, B is three times as likely as C, and C and D are equally likely to be elected. What is the probability that B is elected?

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B6.
Find the area of the square circumscribed about the circle $x^2 - 2x + y^2 = 8$.

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B7.
How many integers x satisfy the equation $|x + 3| + |x - 3| = 6$?

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B8.
Simplify $\left(\sqrt{4 - \sqrt{12}} - \sqrt{4 + \sqrt{12}}\right)^2$.

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B9.
When the product $(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$ is expanded, what is the coefficient of x^{14}?

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B10.
Suppose $\dfrac{x^2 - 1}{x^{3/2} + x + x^{1/2} + 1} = 5$.

$x = ?$

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B11.
When the number n is written in base 4, its digits are AB.
When n is written in base 7, its digits are BA.
What number is n?

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B12.
A parallelogram has vertex $(6, 5)$ and a diagonal with endpoints $(1, 3)$ and $(10, 10)$. Find the fourth vertex.

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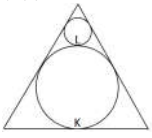
B13. Each side of an equilateral triangle has length 2. Circle K is tangent to all 3 sides of the triangle. Circle L is tangent to 2 sides of the triangle and circle K. What is the diameter of circle L?



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B14.
Suppose that, for all positive real numbers x,

$$f^{-1}(2017x) = \frac{1}{1+x}.$$

Evaluate $f(2017)$.

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B15. An integer greater than 1 is a *prime number* if its only divisors are 1 and itself.
If n is a positive integer, then $n! = 1 \cdot 2 \cdots (n-1) \cdot n$.
How many prime numbers are there among the 27 numbers

$25!+2,\ 25!+3,\ 25!+4,\ \cdots,\ 25!+27,\ 25!+28$?

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B16.
How many ways are there to arrange the letters in the word GENESEE?

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B17.
The numbers $2017k + 1$, $2018k - 2$, and $2017k + 10$ are in arithmetic progression. Find k.

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B18.
Find the smallest positive number k for which the polynomial

$$q(x) = x^2 + 2kx + (4k + 5)$$

has a real root.

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B19. A hyperbola has vertices at $(3, 0)$ and $(-3, 0)$ and foci at $(5, 0)$ and $(-5, 0)$ respectively. If $(3, b)$ is a point on the asymptote of the hyperbola with a positive slope, find the value of b.

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B20.
Of the integers from 1 to 1000, how many contain the digit 7 at least once in their usual decimal representations?

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B21. Four mathematicians, A,B,C and D are seated at a table in a bar. The server waits on them. The conversation goes:

Server: Would each of you like a beer? A: I don't know.
Server: Would each of you like a beer? B: I don't know.
Server: Would each of you like a beer? C: I don't know.
Server: Would each of you like a beer? D: Yes.

The server, who has also studied mathematics, understands the reason for each of the answers.

How many beers does he serve?

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B22. The five-digit number $vwxyz$ has the property that the 6-digit number $vwxyz1$ is exactly 3 times the 6-digit number $1vwxyz$. What number is $vwxyz$?

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B23.
Each face of a regular tetrahedron has area $\sqrt{3}$. What is the volume of the tetrahedron?

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B24.
Suppose that $f(x) = 3x - a$, for all real numbers x.
If $f(f(a)) = 10$, find a.

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B25.
If $f(x) + f(x+1) = x + 1$ and $f(2011) = 5$, find $f(2017)$.

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B26.
The graph of the function

$$f(x) = 2x^2 - 7x + 5c$$

intersects the x-axis at exactly one point. Find the value of c.

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B27.
One root of the equation

$$x^4 - 3x^3 + 11x^2 - 27x + 18 = 0$$

is $3i$ (where $i^2 = -1$). What is the product of the real roots of this equation?

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B28.
Solve for x:

$$x + 10\sqrt{x+3} = 116.$$

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B29. Consider these two columns of numbers.

123456	1
123450	21
123400	321
123000	4321
120000	54321
100000	654321

Which has the greater sum?
(a) The column of numbers on the left.
(b) The column of numbers on the right.
(c) The sums of the two columns are equal.

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B30.
The points $(5, 0)$ and $(-5, 0)$ lie on perpendicular lines through the point $(3, k)$, where k is a positive number.

Find k.

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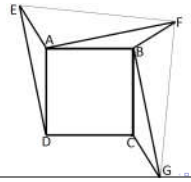
B31. In the diagram below, $ABCD$ is a square, and the triangles ADE, BAF and CBG are all congruent. What is the measure of $\angle EFG$?



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B32. Let a, b, c be the three roots of the equation $x^3 - 3x^2 + 2x + 7 = 0$. What is the value of

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ac}?$$

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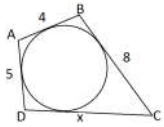
B33.



A circle is inscribed in quadrilateral $ABCD$, which has sides of length, 5,4,8, as shown. What is the length of the remaining side?

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B34.
Add all of the digits of all of the integers from 1 to 999. What is the sum?

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B35.
How many planes contain two of the edges of a cube but none of the faces?

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B36.
Two rectangles with integer sides have diagonals of the same length, $\sqrt{85}$, but they have different areas.

Find the sum of their areas.

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B37.
Solve for x:

$$8x + 16 = 2x^{-3/4} + x^{1/4}$$

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B38.

$$x + y = 3 \quad \text{and} \quad xy = 1.$$

$$x^3 + y^3 = \ ?$$

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B39. The number $a + \sqrt{7}$ is a root of the polynomial $p(x) = x^2 + 6x + b$, where a and b are both integers.
Find a.

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B40.
Evaluate $2017^2 - 2018 \cdot 2016$.