2018 MATH FIELD DAY
HUDDLE PROBLEMS AND SOLUTIONS

Problem 1. How many integers n are there such that $\sqrt{n}$, rounded to the nearest integer, is 2018?

Solution. We are counting integers whose square roots are between 2017.5 and 2018.5. Equivalently, since

$$2017.5^2 = \left(2018 - \frac{1}{2}\right)^2 = 2018^2 - 2018 + \frac{1}{4}$$

and

$$2018.5^2 = \left(2018 + \frac{1}{2}\right)^2 = 2018^2 + 2018 + \frac{1}{4}$$

we need to count the number of integers between $2018^2 - 2017$ and $2018^2 + 2018$, inclusive, which gives a total of $2 \cdot 2018 = \boxed{4036.}$ □

Problem 2. The following expression evaluates to an integer. Find that integer.

$$\left(\sqrt{21 + 5\sqrt{17}} \;+\; \sqrt{21 - 5\sqrt{17}}\right)^2$$

Solution. First, distributing the square gives

$$\begin{aligned}\left(\sqrt{21 + 5\sqrt{17}} \;+\; \sqrt{21 - 5\sqrt{17}}\right)^2 &= (21 + 5\sqrt{17}) + (21 - 5\sqrt{17}) + 2\sqrt{(21 + 5\sqrt{17})(21 - 5\sqrt{17})} \\ &= 42 + 2\sqrt{21^2 - 5 \cdot 17^2} \\ &= 42 + 2\sqrt{16} \\ &= \boxed{50.}\end{aligned}$$

□

Problem 3. There are two positive integers k for which

$$\sqrt{k^2 + k + 75}$$

is an integer. Find the product of these two values for k.

Solution. Since $\sqrt{k^2 + k + 75} > k$, if k is one of the desired values, then we can write $\sqrt{k^2 + k + 75} = k + n$ for some integer $n > 0$. Squaring both sides and then solving for k gives

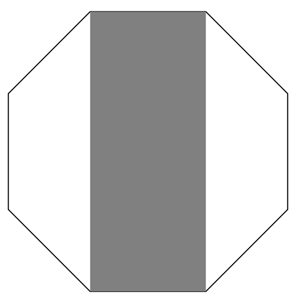
$$\begin{aligned}(k+n)^2 &= k^2 + k + 75 \\ 2kn + n^2 &= k + 75 \\ k &= \frac{75 - n^2}{2n - 1}.\end{aligned}$$

If $n \geq 9$ we would have $k < 0$, so it suffices to check the values of n from 1 to 8, which give respectively:

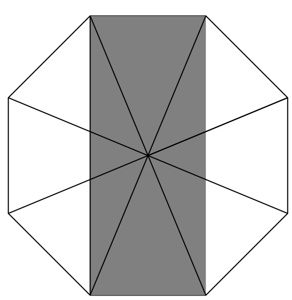
$$k = 74, \frac{71}{3}, \frac{66}{5}, \frac{59}{7}, \frac{50}{9}, \frac{39}{11}, 2, \frac{11}{5}.$$

The product of the two integer solutions, 2 and 74, is $\boxed{148.}$ □

Problem 4. The shaded rectangle constitutes what fraction of the regular octagon's total area?



Solution. Consider dividing the octagon into 8 congruent wedges as shown. The shaded rectangle is comprised of two full such triangles, plus two more triangles with the same area as the first two (as they have twice the base but half the height). In total, the rectangle has the same area as four of the initial wedges, so has $\boxed{\frac{1}{2}}$ of the area of the initial octagon.



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