2013 HUDDLE PROBLEMS
WITH SOLUTIONS

Problem 1. For a real number x, the notation $\lfloor x \rfloor$ represents the result of rounding x down to the nearest integer. For example, $\lfloor 3.9 \rfloor = 3$ and $\lfloor 7 \rfloor = 7$. Evaluate

$$\lfloor \log_{10}(1) \rfloor + \lfloor \log_{10}(2) \rfloor + \lfloor \log_{10}(3) \rfloor + \cdots + \lfloor \log_{10}(999) \rfloor.$$

Solution. We have

$$\lfloor \log_{10}(k) \rfloor = \begin{cases} 0 & \text{if } 1 \leq k \leq 9 \\ 1 & \text{if } 10 \leq k \leq 99 \\ 2 & \text{if } 100 \leq k \leq 999 \end{cases}$$

Now the sum in question contains 9 zeros, 90 ones, and 900 twos, for a total sum of

$$9 \cdot 0 + 90 \cdot 1 + 900 \cdot 2 = \boxed{1890.}$$

□

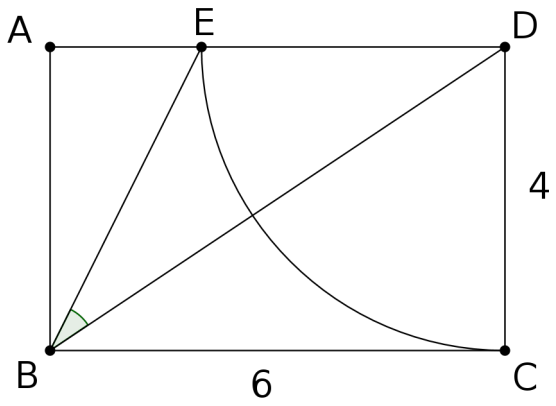
Problem 2. Bobbo, Carly, and Drake make the following statements:

Bobbo: Drake is telling the truth and the number 317 is prime.
Carly : If Bobbo is lying, then $\pi > \frac{22}{7}$.
Drake: Exactly one of Bobbo and Carly is lying.

How many people are lying?

Solution. Note that $\pi \approx 3.1415$ is less than $\frac{22}{7} \approx 3.1428$. If Carly is telling the truth, then since $\pi < \frac{22}{7}$, it must be that Bobbo is telling the truth. But this implies that Drake is also telling the truth, which is impossible, since in this scenario neither Bobbo nor Carly are lying. With this contradiction we learn that Carly is necessarily lying. Negating her implication, we learn that Bobbo is lying. From this we learn that Drake is lying (since 317 *is* indeed prime), so all $\boxed{3}$ of them are lying. Tsk tsk. □

Problem 3. In the diagram below, $ABCD$ is a rectangle, and the arc from E to C is a quarter of a circle centered at D. Find $\tan(\angle EBD)$.



Solution. We note that $DE = 4$ and so $AE = 2$, and so the right triangles $\triangle EAB$ and $\triangle DAB$ give

$$\tan(\angle ABE) = \frac{2}{4} = \frac{1}{2} \qquad \text{and} \qquad \tan(\angle ABD) = \frac{6}{4} = \frac{3}{2}.$$

By the tangent difference formula, we get

$$\tan(\angle EBD) = \tan(\angle ABD - \angle ABE) = \frac{\tan(\angle ABD) - \tan(\angle ABE)}{1 + \tan(\angle ABD)\tan(\angle ABE)} = \frac{\frac{3}{2} - \frac{1}{2}}{1 + \frac{1}{2} \cdot \frac{3}{2}} = \frac{1}{\frac{7}{4}} = \boxed{\frac{4}{7}.}$$

□

Problem 4. The cube of a positive integer x is a 5-digit integer whose first and last digits are 7. Find x.

Solution. Since $40^3 = 64,000$ and $50^3 = 125,000$, we must have x between 40 and 50. Since 3 is the only digit whose cube has ones digit 7, the only possible value for x is $\boxed{43.}$ In fact, $43^3 = 79,507$. □