Huddle 2001: Solutions

1. The equation $x^2 - 14x + 3 = 0$ has solutions r and s. Compute and simplify $1/r + 1/s$.

Solution: We know that $x^2 - 14x + 3 = (x - r)(x - s) = x^2 - (r + s)x + rs$, so $r + s = 14$ and $rs = 3$. Therefore $1/r + 1/s = (r + s)/rs = 14/3$.

2. How many ways are there to arrange the letters in the word BADGER so that BAD does not appear in the arrangement? For example, we count the arrangements BAEDRG and BDAEGR, but not EGBADR.

Solution: To count the number of arrangements which *do* contain BAD, think of BAD as a single symbol. Then there are 4 symbols to arrange (BAD,G,E,R), and so 4!=24 arrangements. There are 6!=720 arrangements of BADGER all together, leaving 720-24=696 which do not contain BAD.

3. Find the three-digit integer such that interchanging the first two digits reduces the number by 270, interchanging the last two digits increases the number by 18, and the sum of the digits is a multiple of 7.

Solution: Suppose the number has digits ABC. The first fact tells us that $(100A+10B+C)-(100B+10A+C) = 90(A-B) = 270$, so $A = B+3$. The second fact tells us that $(100A+10C+B)-(100A+10B+C) = 9(C-B) = 18$, so $C = B + 2$. Therefore $A + B + C = 3B + 5$ is a multiple of 7; this forces $B = 3$. The number is 635.

4. Points A, B, C, D in space do not all lie on the same plane. How many planes in space are the same distance from each of these 4 points?

Solution: You can group the points into two sets either as a set of 3 and a set of 1 (in 4 different ways) or as 2 sets of 2 (in 3 different ways).

Suppose, for example, you have grouped them into set $\{A, B, C\}$ and set $\{D\}$. Then points A, B, C determine a plane. Translate that plane parallel to itself toward D until its distance from D is equal to its distance from the plane ABC; there is exactly one such plane.

If you have grouped them into, say, $\{A, B\}$ and $\{C, D\}$, start with the plane which contains line AB and is parallel to line CD. As before, translate the plane parallel to itself toward CD until its distance from CD is equal to its distance from AB; again, there is exactly one such plane.

There is one plane for each way of grouping the points, a total of 7 planes.