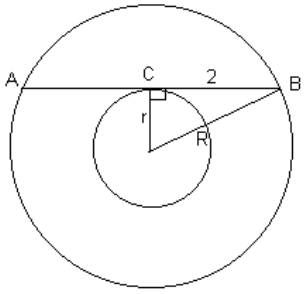
# Huddle Solutions 2003

1. $AB$ is a chord of the outer circle and a tangent to the inner circle. If the circles have the same center and $AB$ has length 4, what is the area of the ring between the two circles?




Solution: Let $C$ be the point where $AB$ touches the inner circle. By the Pythagorean Theorem, the area of the ring is $\pi(R^2 - r^2) = \pi \cdot 2^2 = 4\pi$.

2. How many positive integers below 1000 are multiples of 4 and 6, but not 9?

Solution: This is the same as counting numbers which are multiples of 12 but not 36. The required number of multiples of 12 is $\lfloor 1000/12 \rfloor = 83$, and the required number of multiples of 36 is $\lfloor 1000/36 \rfloor = 27$, so the answer is $83 - 27 = 56$.

3. Define the operation $x * y = \dfrac{x+y}{1+xy}$. Evaluate $1 * (2 * (3 * (4 * 5)))$.

Solution: For any number $z \neq -1$, $1 * z = \dfrac{1+z}{1+z} = 1$. The answer is 1.

4. How many ways are there to express 10 as the sum of positive integers? For example, there are 8 ways to express 4, namely 4, 3+1, 2+2, 2+1+1, 1+3, 1+2+1, 1+1+2, 1+1+1+1.

Solution: Write 10 X's in a row, and place dividers between them to divide them into groups; each grouping of the X'x corresponds to one possible expression. For example,

X | XXXX | XX | XXX  corresponds to 10=1+4+2+3, while

XXX | XXXX | XXX  corresponds to 10=3+4+3.

With 10 X's, there are 9 spaces for dividers. Choosing whether or not to put a divider in each space gives $2^9 = 512$ ways to place the dividers. The answer is 512.