# Huddle 2005 Solutions

1. When a certain four-digit decimal number is multiplied by 4, its digits are reversed. What is this number?

Solution: Suppose $ABCD \times 4 = DCBA$. $4A$ must be have only one digit, so $A = 1$ or 2. But $4D$ ends in the digit A, so A must be even; $A = 2$, and $D = 8$. So now we have $2BC8 \times 4 = 8CB2$.

Therefore $4(10B + C) + 3 = 10C + B$, which reduces to $13B + 1 = 2C$. The only solution is $B = 1$, $C = 7$.

The required number is $ABCD = 2178$. The equation is $2178 \times 4 = 8712$.

2. Consider the four points A, B, C, D in the plane as shown below. Find the coordinates of the point P that minimizes the total distance $PA + PB + PC + PD$.
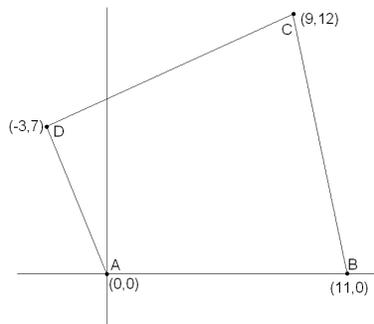
Solution: $PA + PC$ is minimized when P is on the line segment AC, and $PB + PD$ is minimized when P is on the line segment BD. Therefore P must be the intersection point of the line AC (whose equation is $4x = 3y$) and the line BD (whose equation is $x + 2y = 11$). The required point is $P = (3, 4)$.

3. If you were to write out all of the integers from 1 to 1,000,000,000 in their usual decimal form, how many times would you write the digit 7?

Solution: Imagine that leading zeros were allowed, so every number from 000,000,000 to 999,999,999 would have exactly nine digits. That means that there would be a total of 9,000,000,000 digits, and one tenth of them would be 7's, so there you would have to write the digit 7 exactly 900,000,000 times.

4. Pat, Quincy and Rose agree to play a series of games of singles tennis. In each game, the person who won plays the person who sat out the previous game. At the end of the series, Pat has played in 12 games and Quincy has played in 25 games. Who lost game number 7?

Solution: There were at least 25 games, and every player plays in at least every other game. The only way for Pat to play in only 12 games is for Pat to play in games $2, 4, 6, \ldots, 24$ and lose them all. This means that there were exactly 25 games and Quincy played in all of them, so he must win every game except perhaps the last one. Rose lost game number 7.