

Huddle 2006 solutions

1. Suppose the grid is filled in so that every row, column, and 2×2 box in double lines contains each of the digits 1,2,3 and 4. Evaluate X .

	1		
2			
3		X	
			4

Solution: The correct diagram is

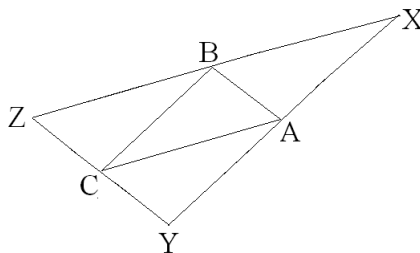
4	1	2	3
2	3	4	1
3	4	1	2
1	2	3	4

$X = 1$.

2. The number $30X0X03$, written in ordinary decimal notation, is a multiple of 13. What digit is X ?

Solution: $3000003 = 13 \cdot 230769 + 6$, and $10100 = 13 \cdot 776 + 12$, so $30X0X03 = 13(230769 + 776X) + 12X + 6$. $12X + 6$ must be a multiple of 13, which is true for $X = 6$.

3. Triangle ABC has area 1. Form a new triangle XYZ by constructing on each vertex of triangle ABC a line parallel to the opposite side. What is the area of triangle XYZ ?



Solution: The parallel lines guarantee that each small triangle has corresponding angles equal to those of triangle ABC . Each small triangle also shares a side with ABC , and so is congruent to ABC . The area of triangle XYZ is 4.

4. Die A is 6-sided, and has the number 1 printed on 5 of its faces, and the number 2 printed on the remaining face. Die B is 4-sided, and has the number X printed on 3 of its faces, and Y on the remaining face. On both dice, each face is equally likely to come up.

When you roll the two dice, the probability is $1/3$ that the sum of the numbers that come up is 5. Evaluate X .

Solution: When you roll the two dice, there are $6 \cdot 4 = 24$ possible outcomes. 15 of them sum to $1 + X$, 3 to $2 + X$, 5 to $1 + Y$, and 1 to $2 + Y$. We need 8 outcomes to sum to 5; this requires that $2 + X = 1 + Y = 5$, so $X = 3, Y = 4$.