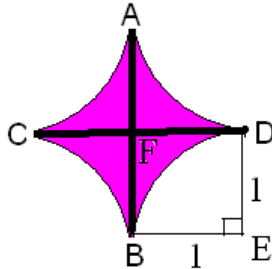


Huddle 2007 Solutions

1. Line segments AB and CD each have length 2, are perpendicular, and bisect each other. Each of the four arcs is an arc of a circle which is tangent to the two line segments. Find the area of the shaded region.



Solution: Let E be the center of the circle containing arc DE . Each of the angles $\angle DFB$, $\angle FDE$ and $\angle FBE$ is a right angle, and $DF = FB$, so quadrilateral $FDEB$ is a square with side 1, and so area 1. DEB is a quarter-circle, so it has area $\pi/4$. The shaded region DFB has area $1 - \pi/4$, so the entire shaded area is $4 - \pi$.

2. The harmonic mean of the positive numbers a, b is defined by the expression $HM(a, b) = \frac{1}{\frac{1}{2}(\frac{1}{a} + \frac{1}{b})}$. If $a + b = 1$, what is the greatest possible value of $a + HM(a, b)$?

Solution: $a + HM(a, b) = a + \frac{2ab}{a+b} = a + 2a(1-a) = -2a^2 + 3a = -2(a^2 - \frac{3}{2}a + \frac{9}{16}) + \frac{9}{8} = -2(a - \frac{3}{4})^2 + \frac{9}{8}$. The largest possible value is $9/8$, which occurs when $a = 3/4$ and $b = 1/4$.

3. Evaluate $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}$.

Solution: Let x be the given number. Then $x^2 = 6 + x$. The only positive root of this equation is $x = 3$.

4. The object of a Sudoku is to fill in the remaining squares so that each of the numbers 1,2,3,4,5 appears in each row, each column, and each of the five regions enclosed in heavy lines. When you solve the Sudoku below, what number goes in the box marked X ?

Solution:

1				3
X				
	5		1	
5			4	

1	4	2	5	3
4	3	1	2	5
3	5	4	1	2
2	1	5	3	4
5	2	3	4	1

$X = 4$.