Huddle 2010 solutions

1. You roll three dice. Take the number on the first die, multiply it by 5, add 7, then double the result. Add the number on the second die, multiply the result by 10, then add the number on the third die. The final result is 401. List, in order, the numbers on the three dice.

Solution: Let x, y, z be the numbers on the three dice, in order. The instructions lead to the equation $((x \cdot 5 + 7) \cdot 2 + y) \cdot 10 + z = 401$, which simplifies to $100x + 10y + z = 261$. The numbers are 2,6,1.

2. Each letter stands for a different digit, to form an ordinary sum.

$$\begin{array}{ccccc} & A & B & C & B \\ + & D & C & E & D \\ \hline C & B & A & B & D \end{array}$$

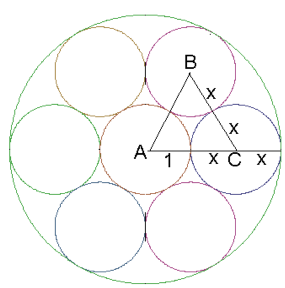
What digit is D ?

Solution: The lead digit in the sum C must be 1, and the one's place shows that $B = 0$. Since there cannot be a carry into the 10's place, $E = 9$. Looking at the 100's place, we see that $A = 2$. Since there is not a carry into the 1000's place, $D = 8$. The complete solution is

$$\begin{array}{ccccc} & 2 & 0 & 1 & 0 \\ + & 8 & 1 & 9 & 8 \\ \hline 1 & 0 & 2 & 0 & 8 \end{array}$$

3. Consider two circles with the same center. The inner circle has radius 1. Between the two circles are 6 more circles, each tangent to the inner circle, the outer circle, and the two adjacent in-between circles. What is the radius of the outer circle?

Solution:



Each line segment marked x is a radius of one of two circles of equal radius. Since $\triangle ABC$ is equilateral, $x = 1$. The radius of the outer circle is 3.

4. Suppose that $F(x, y) = (-y, x)$. Evaluate $F(F(\cdots F(F(20, 10)) \cdots))$; the function F is applied 2010 times.

Solution: Define F^k to be the result of applying F k times. Then

$F^1(x, y) = (-y, x)$; $F^2(x, y) = (-x, -y)$; $F^3(x, y) = (y, -x)$; $F^4(x, y) = (x, y)$,

so $F^k(x, y)$ repeats every 4^{th} iteration. Since 2010 leaves a remainder of 2 when divided by 4, $F^{2010}(20, 10) = F^2(20, 10) = (-20, -10)$.