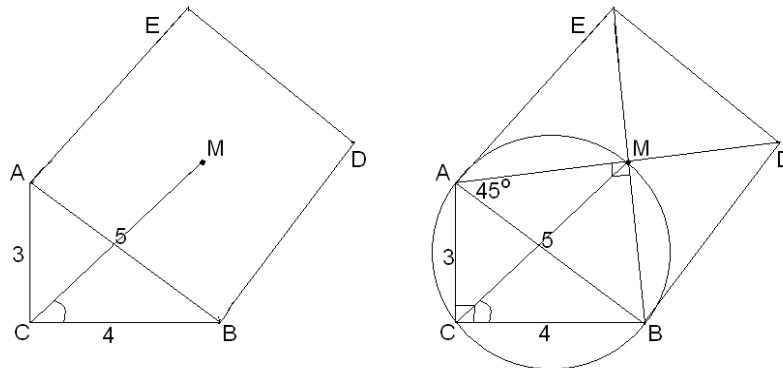


Huddle 2011 – Solutions

1. In the diagram, $AC = 3$, $BC = 4$, $AB = 5$, and $ABDE$ is a square whose center is M . Find the measure of $\angle MCB$.

Solution: Since $\angle ACB = \angle AMB = 90^\circ$, the quadrilateral $ACBM$ can be inscribed in a circle. It follows that $\angle MCB = \angle MAB = 45^\circ$.



2. If you write out the numbers 2^{2011} and 5^{2011} in the usual decimal notation, how many total digits will you write?

Solution: If 2^{2011} has M digits and 5^{2011} has N digits, then $10^{M-1} < 2^{2011} < 10^M$ and $10^{N-1} < 5^{2011} < 10^N$. Therefore $10^{M+N-2} < 10^{2011} < 10^{M+N}$, so $M + N = 2012$.

3. Evaluate $\cos^2(10^\circ) + \cos^2(20^\circ) + \dots + \cos^2(90^\circ)$.

Solution: $\cos^2 x + \cos^2(90^\circ - x) = \cos^2 x + \sin^2 x = 1$. The cosines in the given sum pair off, except for $\cos(90^\circ) = 0$. The sum is 4.

4. In the diagram, $ABCD$ is a square in the (x,y) -plane with side AB horizontal and side BC vertical. Points E, F trisect side AB and points G, H trisect side BC . Points K, L trisect FG and points M, N trisect GH . What is the slope of the line on points L and M ?

Solution: To avoid fractions, let A be the origin, $B = (9, 0)$ and $C = (0, 9)$. The points have coordinates as labeled. The slope of $L = (8, 2)$ and $M = (9, 4)$ is 2.

