

Leapfrog 2003

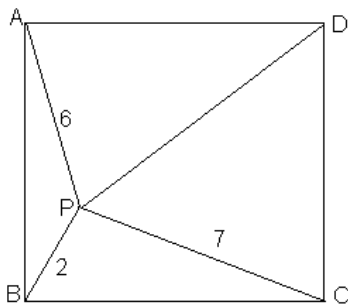
1. ABC is a right triangle with hypotenuse 10 and inscribed circle of radius 2. Evaluate the sum of the lengths of the two legs.

2. Evaluate the product $\sin \frac{\pi}{48} \cdot \cos \frac{\pi}{48} \cdot \cos \frac{\pi}{24} \cdot \cos \frac{\pi}{12}$.

3. How many subsets of the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ contain at least one pair of numbers x, y such that $x + y = 11$?

4. Suppose that w, x, y, z are all greater than or equal to 0, and that $6w + 4x + 3y + 2z = 1$. What is the greatest possible value of $7w + 6x + 5y + 3z$?

5. The point P , inside rectangle $ABCD$, is at distance 6, 2, and 7 from vertices A , B , and C , respectively. Find the distance from P to D .



6. Larry is 5 times as old as Moe was when Curly was twice as old as Moe will be when Larry is one year younger than Curly is now. At present, everyone's age is a whole number, Larry's age is a perfect square, and his elder brother Curly's age is a perfect cube. How old is Moe?

7. A function f whose domain and range are both the set of positive integers, has the properties

$$f(1) = 1, \quad f(2n) = f(n) \quad \text{and} \quad f(2n + 1) = f(2n) + 1.$$

Evaluate $f(2003)$.

8. There are four suspects in a robbery, Pinky, Stuffy, Rusty and The Professor. Each makes a statement.

Pinky says: The Professor is guilty.

Stuffy says: Pinky is innocent.

Rusty says: Either Pinky or Stuffy is innocent.

The Professor says: If Rusty is guilty, then so is Pinky.

Guilty people always lie, innocent people never do. Name all of the culprits.