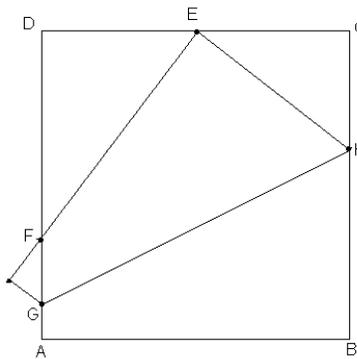# Leap Frog 2004

1. Each side of square $ABCD$ has length 1. The square is folded along crease GH in such a way that vertex B is placed at the midpoint E of side CD. The folding causes side AB to meet side CD at point F. What is the distance from D to F?

2. A polynomial $P(x)$ satisfies the identity $x^5 - \dfrac{1}{x^5} = P\left(x - \dfrac{1}{x}\right)$. Write out $P(x)$ with terms in order of decreasing degree.

3. Line up 10 apples in a row. A set of these apples is called "unfriendly" if it contains no two adjacent apples. How many unfriendly sets of 3 apples are there?

4. Assuming the pattern in this triangle of numbers is continued on, what will the sum of the numbers in the tenth row be?

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 3 & & 5 & & \\
 & 7 & & 9 & & 11 & \\
13 & & 15 & & 17 & & 19
\end{array}
$$

5. The sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. What is length of the shortest side?

6. HATBOX is a 6-digit number – each letter represents a different decimal digit. Given that
$$9 \cdot \text{HATBOX} \; = \; 4 \cdot \text{BOXHAT},$$
what number is HATBOX?

7. A *lattice point* is a point (x, y) in the Cartesian plane where x and y are both integers. A rectangle R has its vertices at lattice points and its sides parallel to the coordinate axes. The sides of R contain a total of 62 lattice points, and the interior contains 100 lattice points. Compute the area of the rectangle R.

8. There are 5 suspects in the Bogarde heist. One is guilty. Each suspect makes a statement.

Emmanuel Ravelli: Either Baravelli or Chicolini did it.

Baravelli: Neither Chicolini nor Tony is guilty.

Chicolini: Emmanuel Ravelli and Baravelli are both lying.

Fiorello: Exactly one of Emmanuel Ravelli and Baravelli is telling the truth.

Tony: Fiorello is wrong.

We know that at least 3 of these statements are true. Who is the guilty party?

(There is no extra credit for identifying the characters in this problem.)