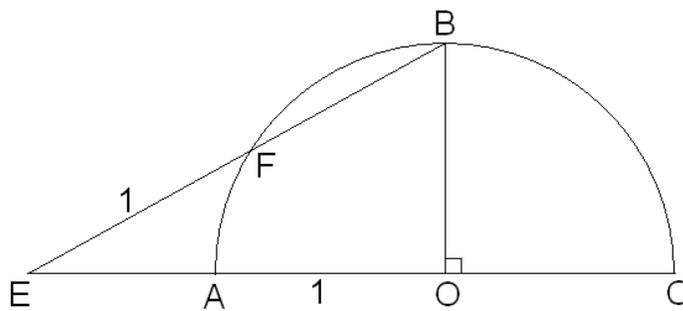


Leap Frog 2005

1. In the figure below, AC is a diameter of the circle with center O and radius 1. Point B lies on the circle, $\angle AOB = 90^\circ$, point E lies outside the circle, line segment EB cuts the circle at point F , and EF has length 1. What is the length of line segment EA ?



2. Suppose p is a polynomial. When p is divided by $x - 1$, the remainder is 3. When p is divided by $x - 3$, the remainder is 1. What is the remainder when p is divided by $(x - 1)(x - 3)$?

3. Express the sum $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} + \frac{1}{143} + \cdots + \frac{1}{9999}$ as a fraction in lowest terms.

4. Twice as many men as women enter a round-robin badminton tournament, yet the ratio of wins by women to wins by men is 7:5. How many women entered the tournament? (A round-robin tournament is one in which every player plays every other player exactly once. There are no ties in badminton.)

5. For each x , define $f(x)$ to be the smallest of x , $2x - 2$, and $10 - x$. What is the the maximum value taken by $f(x)$ over all real numbers x ?

6. Evaluate the infinite sum $\sum_{n=1}^{\infty} \frac{n}{2^n} = \frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \dots$.

7. Each edge of a regular tetrahedron has length one. What is the shortest distance between points on two non-adjacent edges?
(Hint: One way to construct a regular tetrahedron is to start with a cube, and choose 4 vertices no two of which share an edge of the cube.)

8. There are four suspects in a crime. Each makes a statement.

Mrs. Rittenhouse: Exactly one of us is guilty.

Mrs. Teasdale: Two of us did it.

Mrs. Claypool: Three of us conspired to commit the crime.

Mrs. Upjohn: All four of us were involved.

Assume that the guilty suspects are lying, the innocent ones are not. Name all of the guilty suspects.