Leap Frog 2006

1. Suppose that $e^x - e^{-x} = \sqrt{5}$. Evaluate $e^x + e^{-x}$.

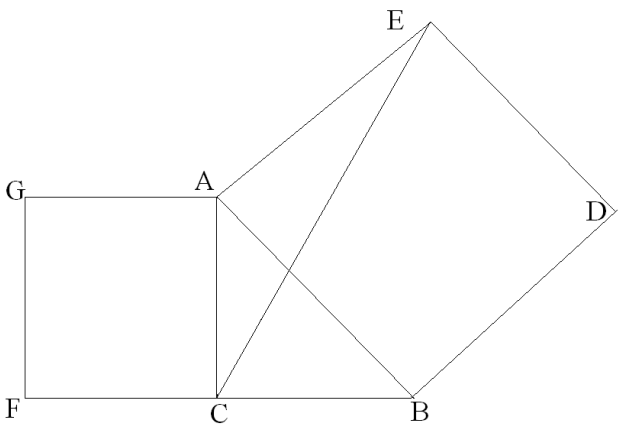
2. How many real-number solutions (x, y) are there to the equation

$$\frac{x^2}{x^2 - 4} + \frac{x^2}{x^2 - 16} = 4.$$

3. Suppose that a, b, c, d are real numbers, and $a^2 + b^2 + c^2 + d^2 = 1$. What is the greatest possible value of $ab + ac + ad + bc + bd + cd$?

4. You are given a right triangle ABC with right angle at C, and squares $ABDE$ and $ACFG$ constructed outwardly on two sides of the triangle. If $AC = 15$ and $BC = 17$, compute the ratio

$$\frac{\text{area of square } ACFG}{\text{area of triangle } ACE}.$$



5. Factor completely the expression $a(b-c)^3 + b(c-a)^3 + c(a-b)^3$.

6. The number 9 can be written as the sum of two or more consecutive positive integers in exactly 2 ways, namely $4+5$ and $2+3+4$. In how many ways can the number 42 be written as the sum of two or more consecutive positive integers?

7. Suppose x and y are real numbers. What is the greatest possible real-number value of the expression $x\sqrt{1-y^2} + y\sqrt{1-x^2}$?

8. There are 5 suspects in a robbery: Homer, Marge, Bart, Lisa, and Maggie. Each makes a statement.

Homer:	Lisa is guilty.
Marge:	Homer and Bart are not both innocent.
Bart:	If Homer is guilty, then so is Marge.
Lisa:	If Marge is guilty, then so is Maggie.
Maggie:	If Lisa is guilty, then so is Bart.

Each innocent suspect told the truth, each guilty suspect lied. Name all of the culprits.