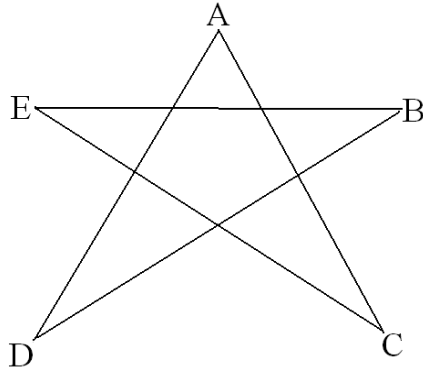


Leapfrog 2007

1. In the figure below, find the sum of angles A, B, C, D and E .



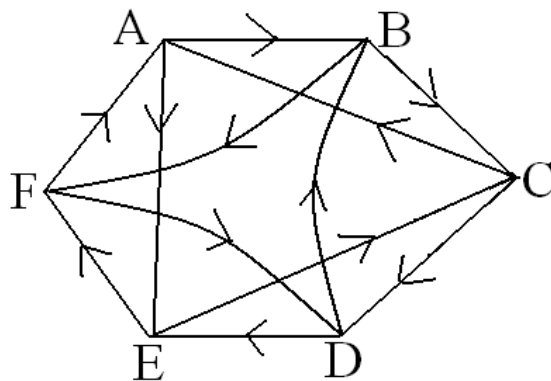
2. Suppose that x, y, z are positive numbers, and

$$(x + y + z)^x = 4, \quad (x + y + z)^y = 2, \quad \text{and} \quad (x + y + z)^z = 32.$$

Evaluate z (as a fraction in lowest terms).

3. Flip a coin 6 times. What is the probability that no 2 consecutive tosses are heads?

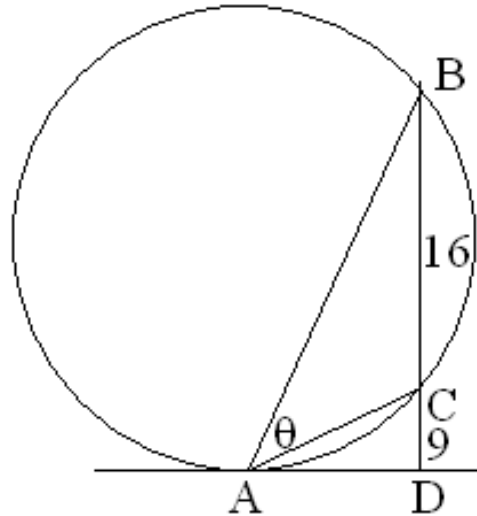
4. An ant moves from labeled point to labeled point along the curves in the direction of the arrows in the diagram below. At time 0, it is at vertex A. At time t (for $t = 0, 1, 2, \dots$), he chooses at random one of the two curves whose arrow leads away from the point he is on, and travels along that curve until he arrives at the next labeled point at time $t + 1$. What is the probability that he is back at point A at time $t = 1000$?



5. Evaluate exactly:

$$1^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) + (1^2+2^2) \left(\frac{1}{2^2} - \frac{1}{3^2} \right) + (1^2+2^2+3^2) \left(\frac{1}{3^2} - \frac{1}{4^2} \right) + (1^2+2^2+3^2+4^2) \left(\frac{1}{4^2} - \frac{1}{5^2} \right) + \dots + (1^2+2^2+3^2+\dots+99^2) \left(\frac{1}{99^2} - \frac{1}{100^2} \right) + (1^2+2^2+3^2+\dots+100^2) \left(\frac{1}{100^2} \right).$$

6. In the figure below, circle \mathcal{K} contains points A, B, C , line AD is tangent to \mathcal{K} , and line BC is perpendicular to line AD . If $BC = 16$ and $CD = 9$, compute $\sin(\angle BAC)$.



7. For any natural number n , $\sigma(n)$ is defined to be the sum of all of the divisors of n . For example, since the divisors of 12 are 1,2,3,4,6,12, $f(12) = 28$.

Suppose p, q, r are three different prime numbers. Find a formula for $\sigma(pqr)$, in factored form.

8. Four notorious thieves named Rickey, Lou, Maury and Ty are accused of theft at El Segundo military base. Each makes a statement.

Rickey : Lou is guilty.

Lou : Maury is guilty.

Maury : Ty is guilty.

Ty : Either Rickey or Maury is guilty, but not both.

Each innocent suspect has told the truth, each guilty suspect has lied. Name the culprits.