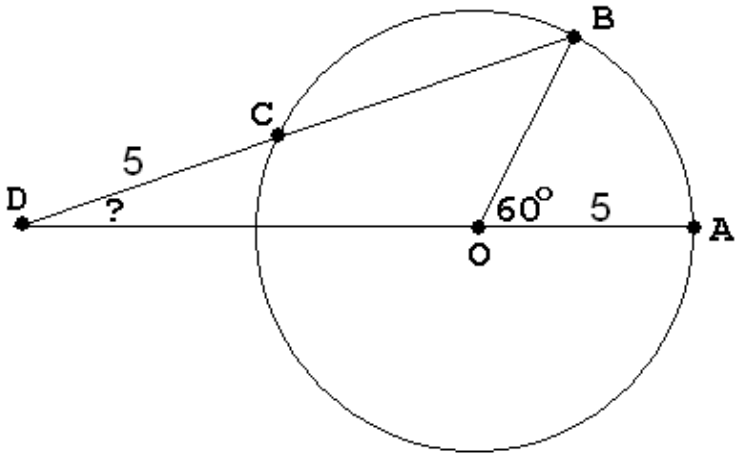
Leapfrog 2008

1. In the diagram, points A, B, C lie on the circle with radius 5 and center O, line BC meets line AO at D, $CD = 5$, and $\angle AOB = 60^o$. Evaluate angle BDA.



2. Three positive integers have a product of 7200 and a sum of 75. Find the largest of the three integers.

3. What is the smallest possible value of

$$f(x) = |\sin x + \cos x| + |\tan x + \cot x| + |\sec x + \csc x| \ ?$$

4. Suppose 3 coins are biased so that they fall heads with probabilities 1/3, 1/5 and 1/7, respectively. Toss all 3 coins; what is the probability you obtain an odd number of heads?

5. Suppose x, y, z are positive integers such that
   (i) $x < y < z$,
   (ii) z has no factors larger than 1 in common with xy, and
   (iii) $(x + y)(x + z)(y + z)$ is a multiple of xyz.

   Find z.

6. Suppose that when you multiply out the polynomial

$$P(x) = (x+1)(x+2)(x+3)\cdots(x+2007)(x+2008),$$

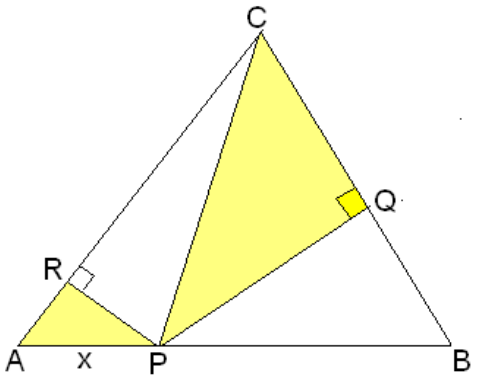
you obtain

$$P(x) = a_{2008}x^{2008}+a_{2007}x^{2007}+a_{2006}x^{2006}+a_{2005}x^{2005}+\cdots+a_3x^3+a_2x^2+a_1x+a_0.$$

Evaluate

$$\frac{a_0 + a_2 + a_4 + \cdots + a_{2004} + a_{2006} + a_{2008}}{a_0 + a_1 + a_2 + \cdots + a_{2006} + a_{2007} + a_{2008}}.$$

7. ABC is an equilateral triangle of side length 1. Points P, Q, R lies on sides AB, BC, CA, respectively, PQ is perpendicular to BC and PR is perpendicular to CA. If AP has length x, find the total shaded area.



8. Three suspects have been arrested for a heinous crime, Bob Rowland, Horatio Jamison, and Frank Wagstaff. Each makes two statements.

Bob Rowland:	I am not guilty. The culprit is bald.
Horatio Jamison:	Rowland is not guilty. The culprit has long hair.
Frank Wagstaff:	I am not guilty. Jamison did it.

No suspect lied twice. Exactly one of the suspects is guilty. Which one?