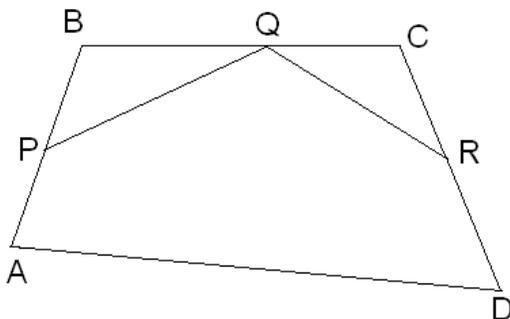# Leap Frog 2009

1. Suppose $ABCD$ is a quadrilateral, and sides AB, BC, CD are of equal length and have midpoints P, Q, R, respectively. PQR is an isosceles triangle, with $PQ = QR = 6$ and $PR = 9$. Find the common length of AB, BC, CD.


2. At a certain pet store there are the same number of dogs, cats and anteaters. However, half of the dogs think they are cats, two thirds of the cats think they are anteaters, and three fourths of the anteaters think they are dogs. (The rest think correctly.) If I choose at random an animal that thinks it is a cat, what is the probability that it is a really a dog?

3. Find the value of x that minimizes the sum

$$\left(\ln\left(\frac{2}{x}\right)\right)^2 + \left(\ln\left(\frac{5}{x}\right)\right)^2 + \left(\ln\left(\frac{8}{x}\right)\right)^2 + \left(\ln\left(\frac{25}{x}\right)\right)^2 + \left(\ln\left(\frac{50}{x}\right)\right)^2.$$

4. For every finite nonempty set S, let $\Pi(S)$ be the product of its elements. Evaluate the sum of $(\Pi(S))^2$ over all nonempty subsets S of $\{1, 2, 3, 4, 5, 6, 7\}$ that contain no consecutive numbers.

5.
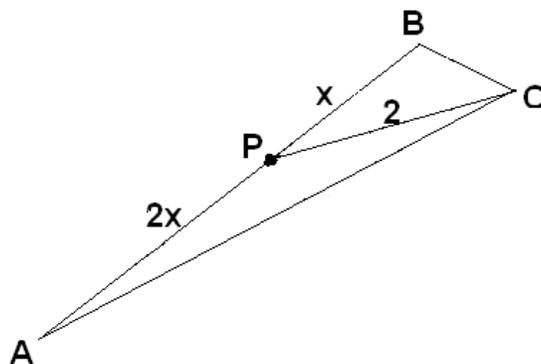
Suppose that ABC is a triangle, that point P on side AB is such that $AP : PB = 2 : 1$, PC has length 2, $\angle BCP = 30^o$ and $\angle PCA = 15^o$. Find the length of AC.

6. For each real number y, define $y^+ = \max\{y, 0\}$.
What is the greatest value taken by the function

$$f(x) = 6(x-1)^+ - 4(x-3)^+ - 5(x-6)^+ + 3(x-10)^+ \quad ?$$

7. List all of the integers m for which the polynomial $p_m(x) = x^3 - mx^2 + mx - (m^2 + 1)$ has at least one integer root.

8. There are five suspects in the Dukenfield caper. Each makes a statement.

L.E. Whipsnade: If Sousé and Criblecoblis are guilty, then so is McGargle.
C.J. Twillie: If McGargle is innocent, then so is Sousé.
E. Sousé: If McGargle is innocent, then so is Criblecoblis.
E.P. McGargle: If Twillie and Sousé are both guilty, then so is Whipsnade.
O. Criblecoblis: Whipsnade is innocent, but Twillie and Sousé are both guilty.

Each guily man lied, and each innocent man told the truth. Name all of the innocent men.