

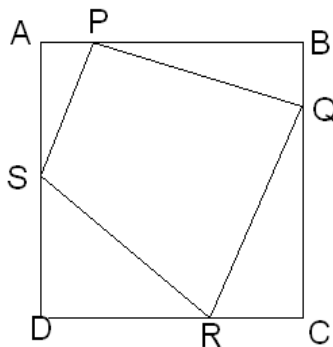
Leap Frog 2010

1. Evaluate $\sec(\tan^{-1}(\cos(\sin^{-1}(1/5))))$.

2. In how many ways can we choose positive integers in nondecreasing order so that their sum is 2010 and the largest and smallest differ by at most 1? That is, in how many ways can we choose positive integers $a_1 \leq a_2 \leq \dots \leq a_n$ such that $a_1 + a_2 + \dots + a_n = 2010$ and $a_n - a_1 \leq 1$?

3. Suppose $0 \leq a \leq 1$, $0 \leq b \leq 1$, $0 \leq c \leq 1$ and $0 \leq d \leq 1$. What is the largest possible value of $S = a + b + c + d - (ab + bc + cd + da)$?

4. Given a square $ABCD$ with each side of length 1, choose points P, Q, R, S on sides AB, BC, CD, DA , respectively. What is the smallest possible value of the perimeter of quadrilateral $PQRS$?



5. Find all pairs of numbers (x, y) with $0 < x < 1$ and $0 < y < 1$ such that $11x + 5y$ and $5x + 2y$ are both integers.

6. Two players, Ann and Bill, play a game. There are 12 chips on a table. Ann picks up a_1 chips, where $1 \leq a_1 \leq 11$. Then Bill picks up a_2 chips, where $1 \leq a_2 \leq a_1$. If there are any left, Ann picks up a_3 chips, where $1 \leq a_3 \leq a_2$. The players continue alternating; on the n^{th} turn, whosever turn it is picks up a_n chips, where $1 \leq a_n \leq a_{n-1}$. The player who takes the last chip wins.

In order to win the game, how many chips should Ann take on her first turn?

7. Find a set of numbers such that

- (i) Each is a power of 3 times a power of 5 (that is, $3^m \cdot 5^n$);
- (ii) none is a multiple of any other; and
- (iii) they sum to 1205.

8. There are 6 suspects in a heist, Cathy A, Anne B, Jane S, Anne C, Kathy H, and Kathy P. Guilty suspects always lie; innocent suspects always tell the truth. Each makes a statement.

Cathy A.: Anne C and I are both guilty.

Anne B: Jane S told me that either she (Jane S) is guilty or I (Anne B) am guilty, but not both.

Jane S: I never said that.

Anne C: My foot itches.

Kathy H: Anne C's foot itches.

Kathy P: Anne C's foot doesn't really itch.

Solve the crime; name all of the guilty suspects.