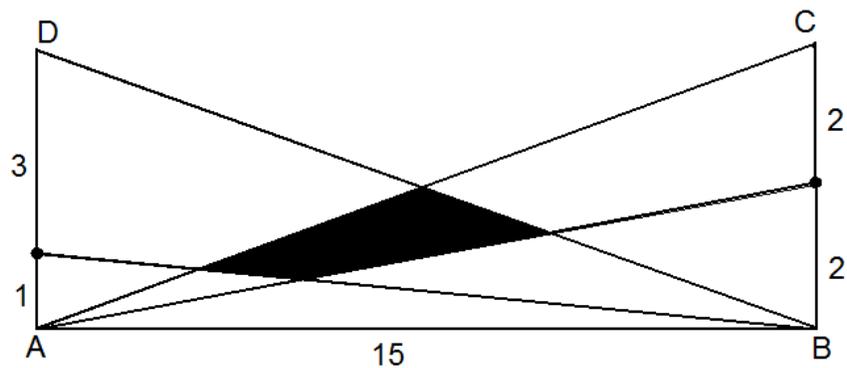


Leapfrog 2011

1. My father is 15 years younger than my grandfather was when I was 95 years younger than my grandfather will be when my father is 50 years older than I am today. How old was my grandfather when my father was born?

2. Four horses race. How many different ways can they finish, if ties are possible?

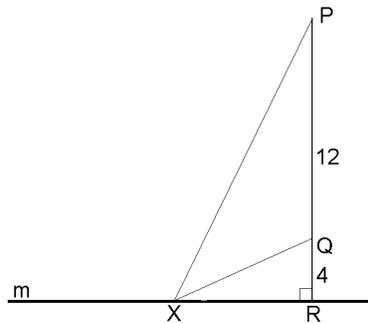
3. Quadrilateral $ABCD$ has right angles at A and B . Find the area of the shaded region.



4. Find the smallest positive integer x such that $(x + y^2)(x^2 + y) = (x + y)^3$ for some positive integer y .

5. If $x + y = 1$ and $x^3 + y^3 = \frac{1}{2}$, evaluate $x^2 + y^2$.

6. You are traveling along road m , looking at a billboard PQ . The billboard is perpendicular to the road. It is 12 meters across, and its nearest edge is 4 meters from the road. How far are you from the point R (the point where the extended billboard would hit the road) when your viewing angle is the best? In terms of the diagram, find the distance XR which maximizes $\angle PXQ$.



7. Suppose that M is a positive integer, and there are exactly 2 positive integers n such that $4n + 1$ is a divisor of $3n + M$. What is the smallest possible value of M ?

8. There are five suspects in a serious heist, the Freak, Matt, Jon, the Mad Bum and the Beard. Each makes a statement.

- | | |
|--------------|---|
| The Freak: | The Mad Bum is innocent or Jon is innocent. |
| Matt: | The Mad Bum and Jon are both innocent. |
| Jon: | One of the Freak and Matt is innocent, and the other is guilty. |
| The Mad Bum: | The Beard and Jon are either both innocent or both guilty. |
| The Beard: | The Freak and the Mad Bum are both innocent |

Innocent suspects never lie, guilty suspects always do. Name all of the culprits.