2012 LEAPFROG PROBLEMS

Problem 1. Find the sum of the 5-digit numbers whose digits consist of one 6, one 8, one 9, and two 1's.

Problem 2. A relay race is held where runner A begins at the origin and runs along a line in the first quadrant until the runner hits the line $x = 1$, at which point he stops and hands the baton to runner B. Runner B runs down the line $x = 1$ until she hits the x-axis, makes a right turn, and returns to the origin along the x-axis. If the entire length of the run was 8 units, how much farther did B run than A?

Problem 3. Find the largest k such that the positive integer

$$\frac{1011(3012^n - 1) - 3011(1012^n - 1)}{1011 \times 3011}$$

ends with at least k zeros for all $n \geq 2$.

Problem 4. An equilateral triangle has one vertex at $(-1, -1)$, and the other two vertices are in the first quadrant on the graph of $y = \frac{1}{x}$. Find the sum of the x-coordinates of the three vertices.

Problem 5. There are unique positive integers $a < b < c$ such that

$$\frac{65}{168} = \frac{1}{a} + \frac{1}{ab} + \frac{1}{abc}.$$

Find $a + b + c$.

Problem 6. For some real value of a, the the polynomial

$$f(x) = x^3 + (3a - 5)x^2 + (a + 3)x - 4a^2 = 0$$

has the unusual property that it has three different positive real roots, each of whose reciprocal is also a root of $f(x)$. Find the largest root of $f(x)$.

Problem 7. Let a, b, c, and d be positive integers such that $n = 2^a 3^b 5^c 7^d$ has the following three properties:

- $36n$ is a fourth power of an integer;
- $\frac{5n}{3}$ is the cube of an integer;
- $\frac{n}{7}$ is an integer.

Find the least possible value of $a + b + c + d$.

Problem 8. If a and b are real numbers such that exactly one of the three equations

$$a^2 + b^2 = 10b, \qquad 11a^2 + b^2 = 100, \qquad 3a + b = 10$$

is false, find a.