2013 LEAPFROG PROBLEMS

Problem 1. Find the largest positive integer n such that the list

$$n^2+1, \quad n^2+2, \quad \ldots, \quad n^2+2013$$

contains the squares of at least two positive integers.

Problem 2. Let $N = 1 \times 2 \times 3 \times \cdots \times 100$. Evaluate

$$\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \cdots + \frac{1}{\log_{100} N}$$

Problem 3. How many digits equal 1 in the result of the following multiplication?

$$\underbrace{666\cdots6}_{\text{2013 sixes}} \times \underbrace{333\cdots3}_{\text{2013 threes}}$$

Problem 4. If a and b are distinct numbers picked at random from the set

$$\{1, 3, 9, 27, 81, \ldots, 3^{2013}\},$$

what is the probability that the polynomial $x^2 + ax + b^2$ has at least one real root?

Problem 5. When expanded, the expression

$$(x+2)^{16}(x^2+1)^8(x^4-1)^4(x^8-2)^2$$

gives a polynomial of degree 64. Find the sum of its coefficients.

Problem 6. Consider the circle of radius 3 whose center is the point $(6,8)$. Find the x-coordinate of the point on this circle that is closest to the origin $(0,0)$.

Problem 7. Let $\alpha = 3.75^\circ$. Evaluate

$$\sin(\alpha)\cos(\alpha)\cos(2\alpha)\cos(4\alpha)\cos(8\alpha)\cos(16\alpha).$$

Problem 8. Simplify the expression

$$\frac{(x+y+z)^3-(x^3+y^3+z^3)}{3(x+y)(x+z)}.$$