

Mad Hatter A 2011

A1. How many points with integer coordinates lie inside the circle $x^2 + y^2 = 7$?

A2. How long is a side of an equilateral triangle that has an inscribed circle of radius 1?

A3. Solve for x: $\sqrt{x+3} - \frac{5}{\sqrt{x+3}} = 4$

A4. $\frac{\cos(x)}{1+\sin(x)} + \frac{\cos(x)}{1-\sin(x)} = 6$

A5. $2x = |x-3|$; $x = ?$

A6. A, B, and C are corners of a cube. AB and BC are diagonals of faces. Find $\angle ABC$ in degrees.

A7. $y = x^2 + k$ intersects $y = 6x$ at just one point. Find k.

A8. $\log_{4x}(x) = -1$, $x > 0$. $x = ?$

A9. Evaluate $\tan^4 x + 2\tan^2 x + 1$ if $\cos x = 1/3$.

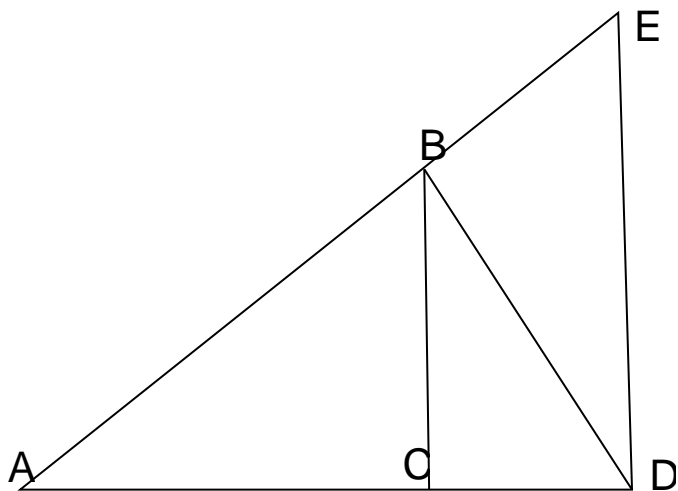
A10. Simplify $\frac{(x+y)^4 - (x-y)^4}{x^2 + y^2}$.

A11. Find the y-coordinate of the point on the y-axis that is the same distance from (1,5) and (3,11).

A12. Simplify $(\cos(10^\circ) + i \sin(10^\circ))^{18}$ for $i^2 = -1$.

A13. Simplify $(\log_2 3)(\log_3 4)$

A14.



$\angle ACB = 90^\circ$, $\angle ABD = 90^\circ$, $\angle ADE = 90^\circ$, $AC = 2$,
 $BC = 1$. $DE = ?$

A15. Find the first n such that $\frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} > 1$.

A16. $f(x) = x^2$, $g(x) = x - 3$, $f(g(x)) - g(f(x)) = 12$.
 $x = ?$

A17. For what number k does $|x^2 - 2| = k$ hold for exactly three numbers x ?

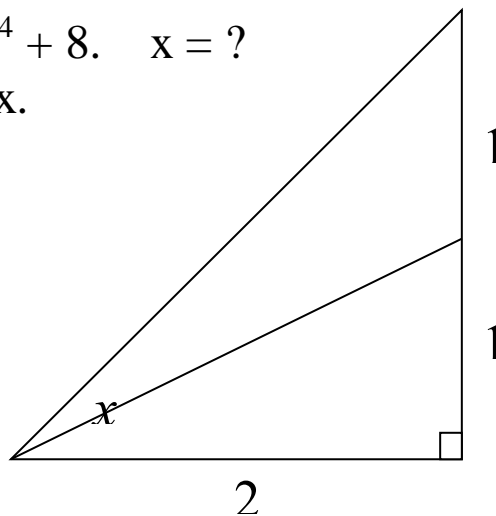
A18. $100^r = 3(10^r) + 4$, $r = \log_{10}(y)$, $y = ?$

A19. How many solutions does $\tan(x) = -x$ have for $-\pi < x < \pi$? x is in radians.

A20. Which of the fractions $3/7$, $4/9$, $5/11$, $6/13$, $7/15$ is the largest?

A21. $x^{3/2} = 7x^{3/4} + 8$. $x = ?$

A 22. Find $\cos^2 x$.



A23. Find the shortest distance from the origin to a point on the graph of $x^2 - 3y^2 + 12 = 0$.

A24. Simplify

$$\sin(\cos^{-1}(503/2011)) - \cos(\sin^{-1}(503/2011)).$$

A25. Find the area enclosed by the graphs of $|x + y| = 2$ and $|x - y| = 2$.

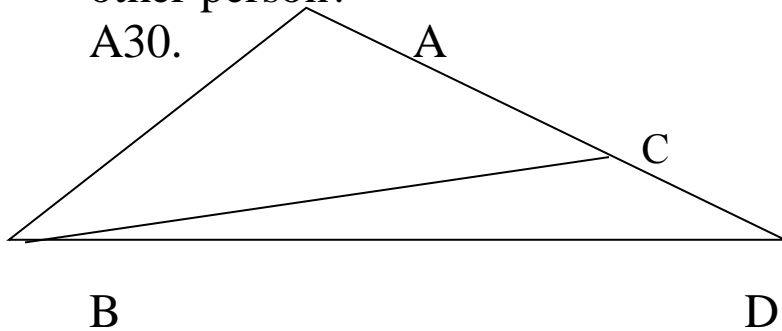
A26. If $f(x) = \frac{2x-3}{x-a}$ and $f(f(x)) = x$ for all $x \neq a$, find a .

A27. Find the greatest slope of a line through (4,5) and a point (x,y) with $y = (9 - x^2)^{1/2}$.

A28. Simplify $\frac{\sin(80^\circ)}{\sin(40^\circ)\sin(50^\circ)}$.

A29. How many ways are there to group 6 people into 3 pairs for handshakes so each person shakes hands with one other person?

A30.



$AB = 5 = AC$, $CD = 1$, $BC = 8$. Find the area of ΔBCD .

A31. $x^2 + 5x + k$ has a root 3 less than k . $k^2 = ?$

$$2011^{\log_{10}(x)} = x^3$$

A32.

$$x = ?$$

A33. Roll two dice. What is the probability that at least one die is greater than 4?

A34. What is the maximum value of $x + y$ for $x^2 + y^2 = 18$?

A35. Set $f(x) = \frac{2x+5}{x-3}$ for $x \neq 3$.

$f^{-1}(x)$ is undefined for what value of x ?

A36. Evaluate $\tan(9^\circ) + \cot(9^\circ) - \sec(9^\circ)\csc(9^\circ)$.

A37. For how many integers x does the inequality

$$\sqrt{(x^2 - 4x)^2} \neq x^2 - 4x \quad \text{hold?}$$

A38. Simplify $\log_4(\log_3(\log_2 8))$

A39. The inequality $\frac{1}{2000x - 2011} < \frac{1}{2000x + 2011}$
holds for how many integers x ?

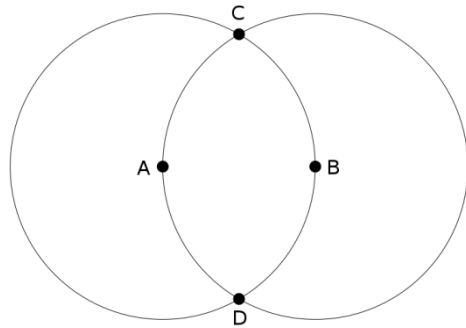
A40. Simplify $\left[\left(\frac{2011x}{x + 2011} \right)^{-1} - 2011^{-1} \right]^{-1}$.

Mad Hatter B 2011

B1. Evaluate $\sqrt{1 + \sqrt{1 + \sqrt{1 + 63}}}$.

B2. Find the least positive multiple of 7 that has remainder 2 on division by 9.

B3.



The circles have centers A and B. Find $\angle CAD$ in degrees.

B4. $100^x(.1)^y = 1000$. $8^x 4^y = 256$. $x = ?$

B5. Solve for x: $\sqrt{x+3} - \frac{5}{\sqrt{x+3}} = 4$

B6. $2x = |x-3|$. $x = ?$

B7. Find the circumference of a circle circumscribed about a regular hexagon of perimeter 6.

B8. Simplify $\frac{(x+y)^4 - (x-y)^4}{x^2 + y^2}$.

B9. Which of the fractions $3/7, 4/9, 5/11, 6/13, 7/15$ is the largest?

B10. $x^{3/2} = 7x^{3/4} + 8$. $x = ?$

B11. How many edges of a cube are not coplanar with a given edge?

B12. Simplify $\left[4x^2 + (1 - x^2)^2\right]^{\frac{1}{2}}$.

B13. Evaluate $-1^2 + 2^2 - 3^2 + 4^2 - \dots - 11^2 + 12^2$.

B14. For what nonzero number a is $x^4 - ax^2 + a$ the square of a polynomial?

B15. Find the y -intercept of the line through $(5,0)$ perpendicular to the line $2y = x$.

B16. The inequality $\frac{1}{2000x - 2011} < \frac{1}{2000x + 2011}$

holds for how many integers x ?

B17. A , B , and C are corners of a cube. AB and BC are diagonals of faces. Find $\angle ABC$ in degrees.

B18. $(1 + 2x^{-1})^{-1} = 3$. $x = ?$

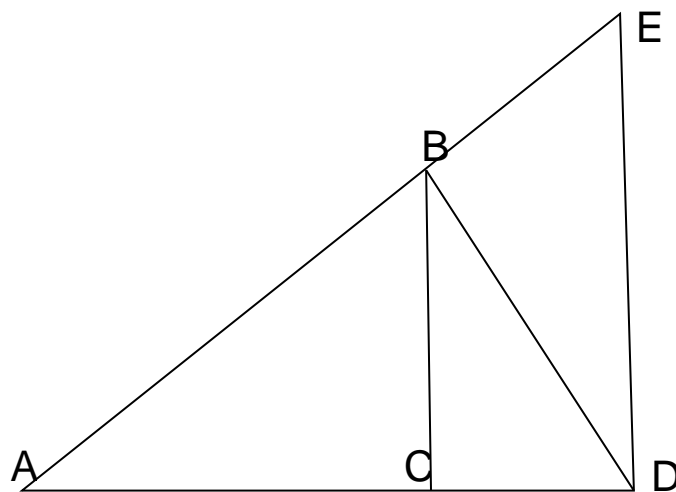
B19. How many positive integers have squares less than 2^{12} ?

B20. $7 + \sqrt{x}$ and $7 - \sqrt{x}$ are reciprocals. $x = ?$

B21. For how many integers x does the inequality

$$\sqrt{(x^2 - 4x)^2} \neq x^2 - 4x \quad \text{hold?}$$

B22.



$$\angle ACB = 90^\circ, \angle ABD = 90^\circ, \angle ADE = 90^\circ, AC = 2,$$

BC = 1. DE = ?

B23. How many ways can 2 girls and 2 boys line up with the boys together?

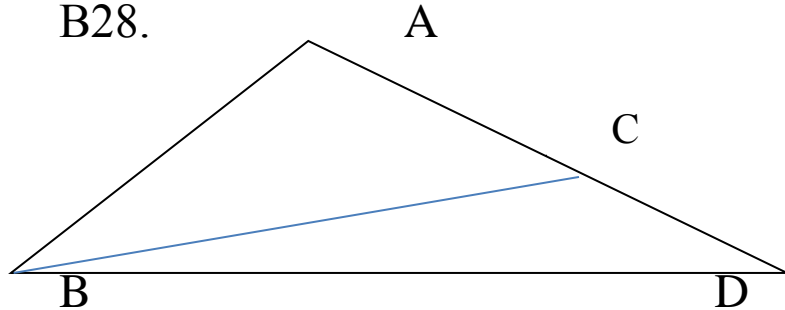
B24. Find the first n such that $\frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} > 1$.

B25. Find the volume of a cylinder with radius π and height π .

B26. $x^2 + 5x + k$ has a root 3 less than k. $k^2 = ?$

B27. Simplify $(1 + \sqrt{2})^3 + (1 - \sqrt{2})^3$.

B28.



AB = 5 = AC, CD = 1, BC = 8. Find the area of $\triangle BCD$.

B29. Roll two dice. What is the probability that at least one die is greater than 4?

B30. $(5 + x^2)^{\frac{1}{2}} = 1 + x$. $X = ?$

B31. If $\frac{r}{2} + \frac{1}{r} = 3$, evaluate $\frac{r^2}{2} + \frac{2}{r^2}$.

B32. Find the smallest positive integer k such that

$\sqrt{28k}$ is an integer.

B33. How long is a side of an equilateral triangle that has an inscribed circle of radius 1?

B34. $y = x^2 + k$ intersects $y = 6x$ at just one point. Find k.

B35. How many ways are there to group 6 people into 3 pairs for handshakes so each person shakes hands with one other person?

B36. $1 - (2 - (3 - (4 - x))) = 2011$. $x = ?$

B37. Simplify $\frac{2011^2 - 1}{2012}$.

B38. Simplify $(2011 \cdot 2013 + 1)^{1/2}$.

B39. 2^{2011} is evenly divisible by how many positive integers? Include 1 and 2^{2011} .

B40. Simplify $\left[\left(\frac{2011x}{x+2011} \right)^{-1} - 2011^{-1} \right]^{-1}$.