Math Field Day 2013
Mad Hatter A

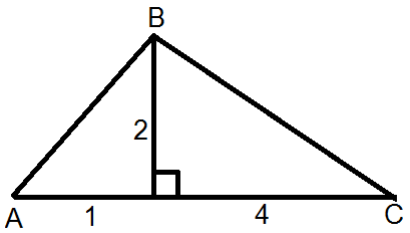
A1

How many points with integer coordinates lie on the segment with endpoints $(0, 0)$ and $(77, 55)$? Include the endpoints.

A2

Find the area of the trapezoid with vertices $(9, 0)$, $(11, 0)$, $(0, 11)$, and $(0, 9)$.

A3



Find $\angle ABC$ in degrees.

A4

For what number c does the point (c, c) lie on perpendicular lines through $(0, 0)$ and $(10, 0)$?

A5

Evaluate

$$f(1) + f(2) + \cdots + f(39),$$

where $f(x) = |x - 20|$.

A6

How many liters of 7% acid solution must be added to 6 liters of 2% solution to get a 5% solution?

A7

How many ways are there to write $800 = ab$ for positive integers a and b such that b/a is an integer?

A8

Simplify

$$\frac{2013^2 - 2013 \times 26 + 169}{10^6}$$

A9

Evaluate

$$[\sin(25^\circ) + \cos(25^\circ)]^2 - \sin(50^\circ)$$

A10

Evaluate

$$\log_{10}(\log_3(7)) + \log_{10}(\log_7(3)).$$

A11

Three dice are rolled. What is the probability that twice the total number of spots that come up on two of them is equal to the number of spots that come up on the remaining one?

A12

How many integers from 100 through 999 have exactly one digit 0?

A13

$x > 0$.
$100^x - 10^x - 10^{x+2} + 100 = 0$.
$x = ?$

A14

Evaluate $r + \frac{5}{r}$ when r is the larger root of

$$x^2 - 9x + 5.$$

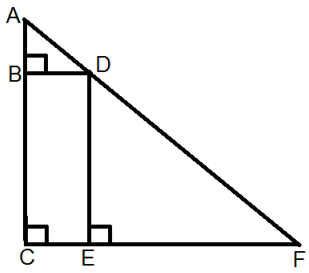
A15

$0 < x < 1$.

$\log(x^{\log(x)}) = 4$, where log is the logarithm base 10.

Write x as a fraction.

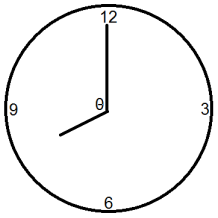
A16



$AC = 10$, $CF = 20$, $BC = 2\ BD$.
Evaluate BD.

A17

The time is 8 o'clock. Let θ be the angle between the minute hand and the hour hand. Evaluate $\sec\theta$.



A18

$-1 < a < 0$. Simplify

$$|a + 2| + \sqrt{a^2} - \log_{a+1}(a + 1).$$

A19

Suppose $a^2 + a = 1$, $b^2 + b = 1$, and $a \neq b$. Find $ab + a + b$.

A20

Suppose $f(x) + f(101 - x) = 1$ for all real numbers x. Evaluate

$$f(1) + f(2) + \cdots + f(100).$$

A21

Suppose $a + b + c = 0$ and $abc \neq 0$. Evaluate

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}.$$

A22

Find the sum of all integers n such that

$$\frac{1}{2} \leq \sin \frac{\pi}{n} \leq 1.$$

A23

Evaluate $\dfrac{3^{2013} - 3^{2012}}{3^{2011} - 3^{2010}}$.

A24

A sports club has 54 members. Among them, 28 members play tennis, 22 play golf, and 10 play both tennis and golf. How many members play neither tennis nor golf?

$$A = \{x, x - y, \frac{1}{xy}\}.$$

A25

$$B = \{0, \frac{1}{4}, 2\}.$$

$$A = B.$$

Evaluate $x + y$.

A26

$$2^2 \cdot 2^3 \cdot \quad \cdots \quad \cdot 2^{10} = 4^x.$$

Evaluate x.

A27

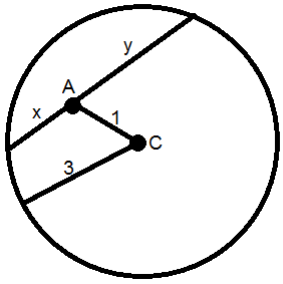
If $x > 0$ and $(x^2 + 4x)^2 - 2(x^2 + 4x) - 15 = 0$, evaluate x.

A28

Pick an integer from 1 through 100 at random. What is the probability that it is either a perfect square or a perfect cube (or both)?

A29

In a circle of radius 3, point A is distance 1 from the center and divides a chord into segments of lengths x and y. Evaluate xy.



A30

Is the sum of 58 consecutive integers

(a) always odd,
(b) always even, or
(c) sometimes odd, sometimes even?

A31

Compute the area of the figure bounded by the graph of the equation

$$2|x| + 3|y| = 12.$$

A32

The 9-digit number

$$123,456,78X$$

is a multiple of 8. What digit is X?

A33

Suppose that, for all x,

$$ax^3 + bx^2 + cx + d = (x+2)^3 + (x-1)^2 + (x-2) + 5.$$

Evaluate d.

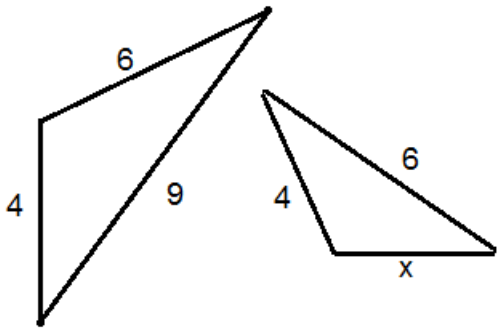
A34

If the points (a, b), $(5a, 2b)$ and $(6a, 12)$ lie on a line and $a \neq 0$, then $b =$?

A35
The average of a, a, b, c, c is 1.
The average of a, b, b, c is 4.
What is the average of a, b, c?

A36

If these triangles are similar but not congruent, then $x =$?



A37

Bill is 4 years older than Ann. When Ann was half Bill's present age, Bill was 2 years younger than Ann is now. How old is Ann today?

A38

Suppose $f(x) = 2x$ and $g(x) = x + 3$ for all x. Simplify

$$f(g(f^{-1}(x))) - x.$$

A39

Simplify

$$\frac{5x^3 + 3x^2 + 5x + 3}{x^3 + 5x^2 + x + 5}.$$

A40

Evaluate $47^2 - 14^2$.

Math Field Day 2013
Mad Hatter B

B1 How many points with integer coordinates lie on the segment with endpoints $(0, 0)$ and $(77, 55)$? Include the endpoints.

B2

Find the area of the trapezoid with vertices $(9, 0)$, $(11, 0)$, $(0, 11)$, and $(0, 9)$.

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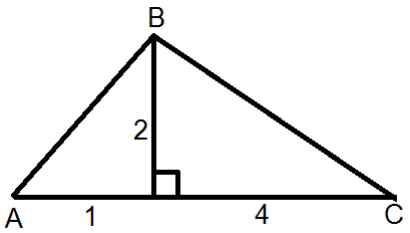

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How many liters of 7% acid solution must be added to 6 liters of 2% solution to get a 5% solution?

B7

How many ways are there to write $800 = ab$ for positive integers a and b such that b/a is an integer?

B8

Simplify

$$\frac{2013^2 - 2013 \times 26 + 169}{10^6}$$

B9

$$\text{If } x + y = 3 \text{ and } x^2 + y^2 = 7,$$

$$\text{then } xy = ?$$

B10

Find the area of the triangle with vertices

$$(0, 0), \quad (3, 1), \quad \text{and} \quad (-1, 3).$$

B11

Three dice are rolled. What is the probability that twice the total number of spots that come up on two of them is equal to the number of spots that come up on the remaining one?

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How many integers from 100 through 999 have exactly one digit 0?

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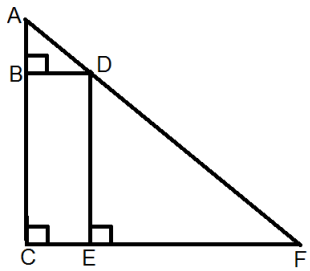

$AC = 10, CF = 20, BC = 2\ BD.$
Evaluate BD.

17

B17

Find the largest possible area of a right triangle with hypotenuse of length 10.

B18

$$\frac{1 + \frac{2x}{x+1}}{2 + \frac{x}{x+1}} = \frac{1 + \frac{3x}{x+2}}{2 + \frac{2x}{x+2}}$$

$x =$?

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For what nonzero number c does

$$x + 8 = cx^{-1}$$

have exactly one solution x?

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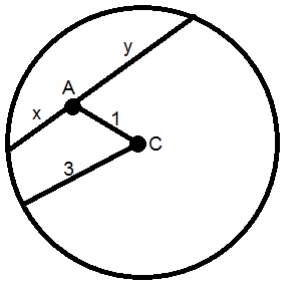

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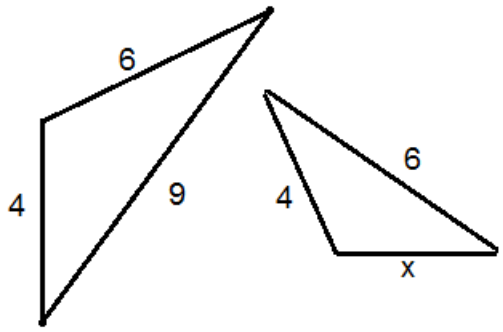

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