MATH FIELD DAY 2012 RELAYS

Relay A

A1. How many ordered pairs \((a, b)\) of positive integers are there such that \(a^2 + b^2 \leq 7\)?

A2. Let \(k\) be the number you receive. Find a positive integer \(x\) such that \(x^2 + x = k^2 - k\).

A3. Let \(k\) be the number you receive. Find the sum of the \(x\)- and \(y\)-intercepts of the line \(kx - 3y = 6\).

A4. Let \(k\) be the number you receive. Find the \(y\)-intercept on the line of slope \(k\) through the point \((3, 2k + 4)\).

A5. Let \(k\) be the number you receive. Find the area of a square with diagonal length \(2k\).

1. 3  2. 2  3. 1  4. 3  5. 18
B1. If $f(x+1) = 2x + 3$, find $f(1)$.

B2. Let $k$ be the number you receive. Find the slope of the line $6x - ky + 3 = 0$.

B3. Let $k$ be the number you receive. Evaluate $1 + \frac{1}{1 + \frac{1}{k}}$.

B4. Let $k$ be the number you receive. If a right triangle has a hypotenuse of length $3k$ and a side of length 3, find the length of the other leg.

B5. Find the radius of the circle $x^2 + y^2 - 2x + 2y = k$.

1. 3 2. 2 3. 5/3 4. 4 5. $\sqrt{6}$
C1. Find the largest prime that divides 1001.

C2. Let \( k \) be the number you receive. If a right triangle has a hypotenuse of length \( k \) and one leg of length 12, find the length of the other leg.

C3. Let \( k \) be the number you receive. If \( a_0 = 0, a_1 = 1 \), and \( a_{n+1} = a_n + a_{n-1} \), find \( a_{2k} \).

C4. Let \( k \) be the number you receive. Find the larger root of \( x^2 - 6x - k = 0 \).

C5. Let \( k \) be the number you receive. Find the sum of the first \( k \) positive integers.

1. 13  
2. 5  
3. 55  
4. 11  
5. 66
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Relay D

D1. How many integers \( x \) with \( 1 \leq x \leq 60 \) are divisible by 2 or 3 (or both)?

D2. Let \( k \) be the number you receive. If an isosceles right triangle has a hypotenuse of length \( \frac{k\sqrt{2}}{4} \), what is the length of each of its legs?

D3. Let \( h \) be the number you receive from the front and \( k \) be the number you receive from the back. If the point \((h, k)\) lies on a circle centered at \((6, 5)\), what is the circle’s radius?

D4. Let \( k \) be the number you receive. If \((x - k)^2 = 9\) and \((x + k)^2 = 1\), find \( x \).

D5. Find the \( y \)-coordinate of the point in Quadrant IV where the line \( x + y = 1 \) intersects the parabola \( x + 4y^2 = 6 \).

1. 40  2. 10  3. 5  4. 2  5. -1