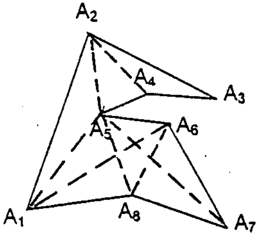
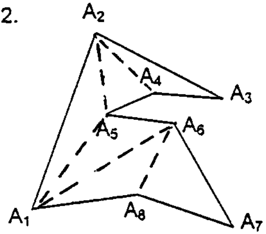
Solutions for the Team Essay

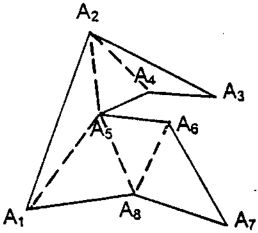
Problem 1.

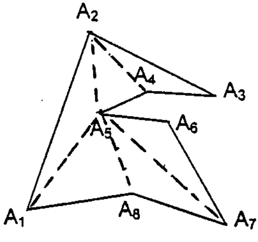


# of diagonals: 7

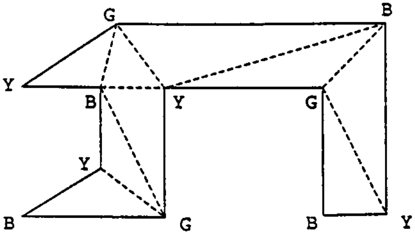
Problem 2.







Problem 3.



4. Because the choice of colors in a 3-coloring doesn't matter, 3-color R so that the endpoints of the diagonal d have the same two colors as in the 3-coloring of Q. Then combine the 3-coloring of Q and R to get a 3-coloring of P (since every triangle in the triangulation of P belongs to either Q or R).

5. Let k be the number of vertices having the color that occurs least often. Since each of the three colors is assigned to at least k vertices, the total number n of vertices is at least 3k. The relation $n \geq 3k$ implies that $k \leq n/3$. It follows that $k \leq [n/3]$, since k is an integer.

6. Since [11/3] = 3, station guards at the three green vertices in the answer to Problem 3 (since green is the only color assigned to just three of the vertices).

7. Set k = [n/3]. Then either n = 3k, n = 3k +1, or n = 3k +2 depending on whether dividing n by 3 leaves remainder 0, 1, or 2. In these three cases, take the polygons in Figures 9a, 9b, or 9c, respectively, that have k spikes. These polygons require at least k = [n/3] guards, since no guard can be stationed to see into more than one spike.

8. Because a 3-gon is a triangle, it is automatically triangulated. Let P be an n-gon with $n \geq 4$ vertices. By assumption, P has a diagonal, which divides P into two polygons Q and R that have fewer than n vertices. By induction, Q and R have triangulations, which combine to give a triangulation of P (since the interiors of Q and R divide the interior of P into nonoverlapping pieces).

9. Let R be the interior of triangle ABC together with all of segment AC except its endpoints A and C. *S* is the set of vertices of P that lie in R.

(a) Segments AB and BC touch exactly two other edges of P, one with endpoint A and another with endpoint C (since $n \geq 4$). Accordingly, if an edge of P touched R, at least one of its endpoints would lie in R, contradicting the assumption that S is empty. Thus, no edge of P touches R, and so R lies entirely inside the polygon P. Therefore, segment AC is a diagonal of P.

(b) If an edge of P touched segment BZ between B and Z,, then at least one endpoint of this edge would lie in R and be farther from line AC than Z is (since segments AB and BC touch other edges of P only at A and C). Because this would contradict the assumption that Z is the vertex in S farthest from line AC, no edge of P touches segment BZ between B and Z. It follows that segment EZ lies entirely inside polygon P and is a diagonal of P.