2014 MATH FIELD DAY
HUDDLE PROBLEMS AND SOLUTIONS

Problem 1. Find the largest natural number n such that

$$n^{2014} < 2014^{1007}.$$

Solution. Taking the 1007-th root of each side, this is equivalent to $n^2 < 2014$. Since $44^2 < 2014$ and $45^2 > 2014$, we conclude that $\boxed{n = 44.}$ □

Problem 2. Evaluate

$$\sqrt{10^{\log_5(10)}} - 10^{\log_5\sqrt{10}}.$$

Solution. The answer is $\boxed{0,}$ as the two terms are equal:

$$\sqrt{10^{\log_5(10)}} = (10^{\log_5(10)})^{1/2} = 10^{\frac{1}{2}\log_5(10)} = 10^{\log_5\sqrt{10}}.$$

□

Problem 3. Find the remainder when the sum

$$1! + 2! + 3! + \cdots + 100!$$

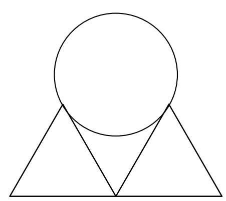
is divided by 15. (Recall that $n! = 1 \times 2 \times 3 \times \cdots \times n$.)

Solution. Each of the terms from 5! on are multiples of 15, so don't contribute to the remainder. We then only have to compute the remainder, when divided by 15, of the terms

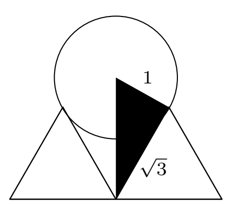
$$1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33.$$

The remainder when 33 is divided by 15 is $\boxed{3.}$ □

Problem 4. A circle is suspended between the tips of two equilateral triangles of side length $\sqrt{3}$, so that the circle is tangent to the edges of the triangles at the vertices. Find the area of the circle.



Solution. Since the circle is tangent to the edge of the triangle, the black triangle below is a 30-60-90 triangle with long leg of length $\sqrt{3}$, and so a short leg of length 1. Since the short leg is also the radius of the circle,



the area of the circle is $\pi(1)^2 = \boxed{\pi.}$ □