2014 MATH FIELD DAY
LEAP FROG PROBLEMS

Problem 1. If m and n are positive integers such that

$$mn + m + n = 2014,$$

what is the largest possible value of m?

Problem 2. Evaluate

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\cdots\left(1 - \frac{1}{2014^2}\right)$$

Problem 3. Suppose that A and B are points in the plane one unit apart. How many points C are there such that $\triangle ABC$ is a right triangle with perimeter $\frac{9}{4}$? (Any of the three vertices of $\triangle ABC$ can be $90°$.)

Problem 4. Evaluate

$$\begin{aligned}&\sqrt{(\cos(107) - \cos(17))^2 + (\sin(107) - \sin(17))^2}\\ +&\sqrt{(\cos(197) - \cos(107))^2 + (\sin(197) - \sin(107))^2}\\ +&\sqrt{(\cos(287) - \cos(197))^2 + (\sin(287) - \sin(197))^2}\\ +&\sqrt{(\cos(17) - \cos(287))^2 + (\sin(17) - \sin(287))^2}\end{aligned}$$

Problem 5. Find x_5 if $x_1 = \log_9(3)$, $x_2 = x_1^{\log_7(2)}$, $x_3 = x_2^{\log_5(8)}$, $x_4 = x_3^{\log_2(5)}$, and $x_5 = x_4^{\log_2(49)}$.

Problem 6. Suppose that 10 fair 8-sided dice (with sides 1, 2, 3, 4, 5, 6, 7, 8) are rolled. Find the probability that the product of the numbers rolled is prime. Write your answer as a reduced fraction in the form

$$\frac{m}{8^n}.$$

Problem 7. Boise is trying to guess the polynomial $x^3 + bx^2 + cx + d$, with integer coefficients b, c, and d, that Salem is thinking of.

Boise: Is $c \leq 20$?
Salem: Yes.
Boise: Does the polynomial factor as $(x - r)^2(x - s)$ for positive integers r and s?
Salem: Yes.
Boise: Could I determine r and s if I knew the value of c?
Salem: No.

Find c.

Problem 8. Suppose that $x > y > 0$ are real numbers satisfying

$$\begin{aligned}(x - y)^{x^2} &= 512\\ (x - y)^{xy} &= 64\\ (x - y)^{y^2} &= 16\end{aligned}$$

Find $(x + y)^2$.