The Debt Cliff: Monte Carlo Simulation, Cash Flow Projection, and Capital Budgeting

Christopher J. Enge
Faculty Advisor: Amy Groseck
School of Management

I. Introduction—The Weakness of Traditional Capital Budgeting Techniques

Perhaps the most important and difficult business decision is whether to commit capital to a risky venture. For example, paying the right price for a new factory leads to success. Paying too much leads to ruin. A typical financial problem involves determining whether a company should proceed with a project or acquire an asset as an investment. Commonly, the firm projects cash flows into the future and compares those cash flows to the initial investment. The company then either computes an internal rate of return (IRR) or net present value (NPV). If the IRR exceeds a pre-determined hurdle rate (frequently but not always the weighted average cost of capital) or the NPV exceeds zero, then the company typically approves the project. If not, the company likely rejects the project.

The problem with this approach is that the traditional method does not model risk in an intuitive way. One set of inputs yields one result and we either accept or reject the investment with absolute certainty without regard to the full possible range of outcomes. Our model, in contrast, enhances a typical multi-year cash flow projection with Monte Carlo Simulation (MCS) and shows an entire range of probable outcomes. The decision maker can evaluate the investment based on both its expected returns and risk.

A simple example illustrates why focusing on a single IRR point may lead to ill-informed decisions. Suppose a company has a required rate of return of 15%. In addition, the company prefers

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1 NPV assumes an initial required interest rate and discounts all cash flows to present value. The sum of the discounted cash flows is NPV. IRR is the interest rate where NPV equals zero.
2 In this paper, we employ IRR rather than NPV. Although NPV has several advantages of IRR, not the least of which is that sometimes there can be more than one IRR for a given set of cash flows, we believe for purposes of this paper IRR is more intuitive. Also, because we use a single initial outflow and a single ending inflow of cash there is only one IRR for each of our cases.
projects with a higher IRR than those with a lower IRR. Suppose further the company has two potential projects, but only enough capital to undertake one or the other. Project 1 (P1) has an IRR of 20% and Project 2 (P2) has an IRR of 17%. Under our rules, we accept P1 because it has the higher IRR.

This simple-minded approach, however, has one big problem. It sometimes leads to the wrong decision. If we consider a distribution of outcomes instead of a single projection a rational investor might very well prefer P2. Suppose P1 has a range of possible outcomes, with a low of -10% and a high of 50%, with an average outcome of 20%. In contrast, P2 has an expected outcome range of 15% and 19% with an expected return of 17%. For most people, the preferable choice is likely the more certain P2 despite the lower average return. However, the projected IRR standing alone does not alert the decision maker to the true differences in risk associated with each investment.

There exist standard financial approaches to accounting for the problem of variability of outcomes. Most commonly, firms address risk subjectively (Lohmann, 1994). The firm could weigh the various risk factors of P1 and its own tolerances to subjectively raise the IRR hurdle rate. Of course, that raises the issue of whether the firm raised the rate by proper amount. Perceived and actual risk may vary leading to poor decisions (Lohmann, 1994).

Another traditional approach is to apply sensitivity analysis (Lohmann, 1994). Here, the firm will make a grid of “what if” scenarios. For example, the firm might generate a grid showing different projected outcomes given differing levels of economic growth. Sensitivity analysis helps the decision maker more fully see the range of possibilities. The worst case and best case scenarios now emerge into plain view. However, there is no probability weight associated with each scenario. If the best case is unlikely, it might encourage a firm to be too aggressive. Conversely, a scary but unlikely worst-case scenario might dissuade a firm from undertaking a project it should.
MCS can help solve some of these problems. What follows is an application of MCS to enhance a standard Excel spreadsheet to enable better financial decision-making. As an example, we present each developmental step in some detail. First, we build a traditional cash flow projection model for a fictional company. Next, we enhance the model to allow it to simulate thousands of scenarios based on distributions of inputs and to simulate situations where the company can default on its debt and go out of business. Lastly, we run the simulation at various levels of debt to see how increased leverage impacts the likelihood of default and average IRR.

II. The Static Cash Flow Model

A. The Static Model

Our model addresses a situation where a firm is considering acquiring a company as an investment. The acquiring firm intends to invest some of its own equity and finance the rest through a leveraged buyout—the target company will borrow the rest of the purchase price. We model the cash flows of the target company so that we can evaluate the investment. We must make our best estimate as to each line item of the cash flow statement.

We assume an initial investment and sale of the firm in five years. The key output is the IRR for the entire five-year period, based on our initial investment and the exit value. We also assume an IRR hurdle rate of 15% to approve the investment. We begin with the company generating $200m in revenue for 2011, the first year. We then assume expenses such as cost of goods sold (86% of sales), selling, general and administrative expenses (3% of revenue), and depreciation (9% of revenue). We then compute earnings before interest, taxes, depreciation and amortization (EBITDA) and earnings before interest and taxes (EBIT). We assume a tax rate of 40%.

The first module of the spreadsheet computes cash flows for the years 2011-2015. We begin with a simple income statement, but because we are interested in cash flows and not taxable income, we make
a series of adjustments to determine our Excess Cash. In the Deficit Funding line, we compute how much the firm would have to borrow to maintain the beginning cash level. If there is excess cash, it is applied to pay down debt early.

The next module, the Debt Calculator, begins with an initial Principal balance. After deducting the mandatory annual payment, we adjust the Principal depending on whether excess cash is available. The ending balance carries over to the next year. The Principal balance is used to compute Interest in the Cash Flow Statement.

The next module computes the value of the company at the end of five years using typical inputs. First, we employ an EBITDA multiplier of 12 to compute the enterprise value. We add the cash balance to the enterprise value to get the total value of the company, i.e. what an investor would pay for the company after five years. Next, we compute the value of the equity by subtracting the debt balance. We can then compute IRR by comparing our initial equity investment to the final equity value. The key output is the final IRR.

We designed the model to allow easy entry of certain assumed inputs. Our inputs include a tax rate, capital structure, initial investment, exit EBITDA multiplier, debt assumptions, and income statement assumptions. The decision maker can change these inputs to see the impact on the final IRR.

**B. Static Model Results and Commentary**

Based on our inputs for this company, if we paid $163m for the equity and borrowed the rest of the $250m purchase price we would expect to sell the equity in 2015 for $408m, yielding an IRR of 20.23%. Our ending debt would be $44m, half of the original $88m debt. Because our IRR hurdle rate is 15%, it is a good investment.
Some limitations apply. One well-recognized problem with the static model is that it rewards higher debt levels with higher IRR as shown in Figure 1.\(^3\) Where the return covers the interest payments, any return above the interest payment is pure profit to the equity holders. At the same time, equity decreases as debt increases. As equity gets smaller our return on equity increases. So long as the return is positive, return on equity approaches infinity as the level of equity approaches zero.\(^4\) The decision maker is therefore tempted to aggressively borrow.

![Figure 1. How IRR Increases in the Static Model as Debt Ratio Increases. If we ignore default risk or increased interest rates at higher debt levels, the static model suggests that the firm can increase IRR simply by increasing its Debt Ratio.](image)

Human nature potentially limits the utility of the traditional static model. By manipulating the assumed inputs, one can make an investment look more attractive. Moreover, a decision maker not involved in the preparation of the model would be frequently hard-pressed to ferret out overly optimistic assumptions. One need look no further than the financial crisis of 2008 to see the results of incorrectly pricing investments.

In addition, by looking at only one scenario the possible dangers do not appear. We do not know the probability of our projected scenario. We also do not know how likely the worst case scenario is or

\(^3\) We compute the level of debt by taking total debt and dividing it by enterprise value (i.e. the total value of the firm). The higher the debt the lower the equity, as Debt + Equity = Enterprise Value. For purposes of this paper, “Debt Ratio” equals Debt / Enterprise Value.

\(^4\) Modigliani, F. and Miller, M. dealt at length with finding an optimal capital structure under different assumptions regarding taxes and default risk. A full discussion of their work goes beyond the scope of this paper. For an introduction to the theory of Modigliani and Miller, see Kolb, 1988, pp. 518-558.
how bad it might be. There is a big difference between a worst-case scenario where the investor might only earn less income and one where the entire investment can be lost.

III. Cash Flow Model with Monte Carlo Simulation

A. Monte Carlo Simulation in Excel

To address these limitations, we modified our model to add uncertainty in the cash flows. Although we continue to use the Excel model, we used the MCS add-ins of Insight.xla 2.0 to model uncertainty in the cash flows.

First, we present some background on MCS. Many times the mathematics of analyzing multiple data sets can prove too difficult even for sophisticated mathematicians (Fatony, Jati, & Fitri, 2010). Moreover, if variables are not independent a solution may be nearly impossible to find without simulation (Fatony et. al., 2010). Simulation techniques, however, allow a decision maker to experiment and observe likely outcomes by repeatedly “throwing the dice.” Instead of employing algebra or calculus to apply probability theory to work out a solution, we simulate the activity and tabulate the results. The results of hundreds or thousands of iterations give us a population of data from which we can derive our statistics and probabilities, solving problems that become too cumbersome for other methods (Tapiero, 2004, p. 163). The long history of MCS and finance includes, for example, Joy and Grube (1981) using MCS to compute negative IRR back when standard tables only allowed for positive returns. Similarly, Sherman and Walters (1997) used MCS to compare various methods of computing rates of return.

MCS is not new to financial modeling (Lohmann, 1994). Kim and Farragher (1981) long ago employed MCS to analyze various capital budgeting practices. What is fairly new, however, is using add-ins in business spreadsheets like Excel to conduct MCSs (Savage, 2003). Because of the power of modern personal computers, MCS is now inexpensive and easy for anyone with access to Excel and programs like @Risk, Risk Solver, Crystal Ball, or Insight.xla (Savage, 2009).
MCS in financial modeling has begun to proliferate in the literature. Fatony et al. (2010) used MCS to compute IRR of simulated projected cash flows from geothermal projects. Despite using Fortran, their outputs are remarkably similar to those set forth in this paper. Similarly, Bieda and Tadeusiewicz (2005) applied Excel-based MCS to project fifteen years of income statements as well as IRR and NPV associated with various equipment investment options. Potluri and Rajan (2010) provide another example by developing distributions from actual price and demand data to simulate in Crystal Ball the returns from power plants in India. MCS will likely become more common in the business literature as case studies have begun to appear designed to teach MCS techniques.

The key to understanding MCS output is to think of probabilities as shapes, not points (Savage, 2009). As opposed to thinking of a cash flow projection as predicting exactly how much money the investment will make, the distribution allows us to see graphically the likely range of possible outcomes. In addition, we can see the probability of surprisingly bad outcomes as well as that of surprisingly good outcomes. Although many outputs are possible, this paper displays those most likely to appeal to business practitioners. We employ easy to read line charts and histograms as opposed to the less familiar ways of displaying data.

MCS allows for many kinds of distributions, but we employ the triangular distribution for our model, having a low, expected, and high value. The apex of the triangle is above the most likely outcome (Hesse, 2000). We believe the triangular distribution is appropriate for many business applications as many business professionals customarily forecast a best case, worst case, and expected scenario (Savage,

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6 Insight.xla 2.0 can generate the S-shaped cumulative probability distributions. It can also generate various three dimensional outputs. Many businesspeople are not familiar with data displayed in these ways. However, a histogram of possible outcomes generally resembles the familiar probability distribution curve, providing an intuitive way for someone unfamiliar with these Monte Carlo statistical methods or cumulative distributions to understand the results.
2003). In practice, however, one would need to test each variable to determine which distribution best represents reality (Potluri & Rajan, 2010).

B. Enhancing the Static Model

We modified our static model so that many of our Assumed Inputs are now represented by distributions. For example, instead of assuming cost of goods sold is always 86% of sales it is now possible for a low value of 81%, most likely value of 83%, and high value of 94%. Our distributions simulate a company with healthy revenue growth prospects but allow for the possibility of growth stalling. We allow for some but not great variability in our other expenses and cash outflows, such as SG&A, depreciation and capital expenditures. In sum, the intent of the distributions is to simulate a typical company enjoying solid growth but still allowing for the possibility of the occasional bad year. Our mean of each distribution mirrors our single point projections in the static model. That is, to compare fairly with the static model each distribution has a mean identical to each single point in the static model.

C. The Default Trigger

1. The default trigger defined

We now take our model another step and account for the risk that the firm could do so badly it defaults on its debt and goes out of business. A critical innovation is the default trigger. Recall that the static model assumes the firm could always borrow to cover a shortfall. That is never true in the real world as for any company there exists some point of distress beyond which lenders will not lend. Our enhanced model sets a level of negative cash flow where we assume that neither a lender nor equity

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7 The mean of a triangular distribution equals (a+b+c) / 3 where a equals the low, b equals the apex or most likely, and c equals the high (Hesse, 2000).
8 Lohmann (1994) in his model provides for the “risk of ruin” but does not explain in detail the conditions that count as “ruin.”
investor would cover a shortfall, leading to the firm’s default. Because in most bankruptcies the equity holders receive nothing, our exit value becomes $0 in the event of default.9

2. Default probability

The model computes the probability the business will fail. We assumed the target firm has access to a line of credit of $15m and that it cannot fund short-term cash shortages in any other way. If our cash flow projection for any year shows the firm borrowing more than $15m, the firm defaults and the exit value is $0. We ran 10,000 iterations at debt levels of 0%, 15%, 30%, 45%, and 60% as shown in Figure 2.

Figure 2 shows that increasing the debt level from 30% to 60% transforms our company from a safe investment to a highly risky investment. It is not clear at this point whether we can generalize that every firm or even many firms would have a tipping point where default rates skyrocket as more debt is added. Our assumed distributions seem reasonable and therefore it seems likely our model does describe reality for many real firms. Potentially useful research could show under what circumstances debt kills a firm.

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9 At the outset, we noted IRR has some weaknesses. Purists might note that there is no way to compute IRR if the end value is zero. However, if instead of using zero we use an end value of a penny, given our beginning investment levels IRR will be almost -100% in the event of default. Our model records -100% if it receives an error in calculating IRR and the error only appears in the event of default and the exit value equals $0.
D. IRR Distributions at Increasing Levels of Debt

Another way to graphically illustrate the same data is to show distributions of IRR at each debt level. At Debt Ratio equals 0%, we achieve a modest 11.7% mean IRR. The low IRR is attributable to the lack of leverage. But the range of outcomes is small with most results between -3% and 28% as shown in Figure 3. If we compare our 11.7% mean IRR to our 15% hurdle rate, we would reject the investment.

At Debt Ratio equals 15%, our mean IRR increases to 15.1%, while the range widens to -10% to 35%. The distribution becomes wider and flatter while moving to the right. Similarly, at Debt Ratio equals 30%, we achieve a slight uptick in average IRR to 16.2%. The range remains similar with most outcomes between -10% and 35%. Note that if we were using a 15% hurdle rate, we would reject the

Figure 2. Probability of Default Plotted Against Debt Ratio. If Debt Ratio is less than 30%, we observe no defaults. However, when Debt Ratio equals 60%, the firm defaults nearly 90% of the time.

Figure 3. Histogram of Distribution of Returns if Debt Ratio = 0%. Horizontal axis shows IRR ranges, vertical axis shows frequency.
investment if it were financed entirely with equity, but accept it at 15% or 30% debt. In other words, we accept the investment because leverage increases the projected IRR, consistent with the pattern seen with the static model.

At Debt Ratio equals 45%, our mean IRR drops to -4.7%, because the firm now defaults about 20% of the time. The investment should now be rejected. The rest of the distribution, however, shifted right. If we escape default, we have a good probability of IRR above 20%, mirroring the spectacular results observed in good economic times. As we increase the Debt Ratio to 60%, mean IRR plummets to -82.5%. However, when we win we win big as most of the positive results exceed 13% and a good number exceed 35% as shown in Figure 4. However, the large probability of default offsets any gain from the spectacularly successful scenarios. In contrast to the static model, we would now reject the highly leveraged scenarios because IRR falls so greatly due to the defaults.

E. The Debt Cliff—Graphing Mean IRR Against Debt Ratio

The next step is to illustrate the change in IRR distribution against debt ratio in a single easy to read chart. If we graph the mean IRRs against Debt Ratio, we find that mean IRR increases up to Debt Ratio equals 30% but then falls off a cliff as shown in Figure 5.
For this firm, at first glance it appears the optimal debt level would be 30% because that level yields the highest average IRR. However, that logic ignores that in practice our estimate of the distributions might be wrong. Moving from 15% to 30% only slightly increases IRR, but moves us much closer to the cliff. In practice, we will not know exactly where the cliff lies. The prudent course, in this case at least, may be to avoid going near the cliff because the small marginal increase in average IRR is not worth the risk. A benefit of this kind of model is that the decision maker can better see the risks involved with a particular capital structure.

V. Conclusion

We set out to demonstrate how MCS can improve investment and capital structure decisions. Traditional IRR and NPV models do not account very well for potential variability in outcome. Our enhanced model provides outputs that decision makers could use to fully appreciate the risk associated with a given investment. By generating easy to read histograms, the model provides information going beyond that of the IRR hurdle rate approach.

A good model should reveal information that would otherwise remain hidden. Our model succeeded by showing that for our fictional company default risk rises swiftly as debt levels increase beyond a certain point. Our model could help investors and lenders recognize the dangers of the debt cliff and take appropriate steps to avoid taking on too much default risk.

Figure 5. The Debt Cliff. Average IRR increases gradually until the Debt Ratio equals 30%, beyond which mean IRR plummets as the Debt Ratio increases to 45% and 60%.
We face a number of limitations even with the MCS model. MCS oftentimes does a good job of modeling general tendencies and trends but fails to reach empirical certainty. For example, it is unlikely the model could accurately determine the exact location of the debt cliff for a given company, but the model could likely give a decision maker a good idea of where debt levels become excessive.

Another limitation is that while MCS allows us to make better estimates of our inputs and provides reasonably good outputs MCS sometimes masks cause and effect. It is difficult, as with any experiment, to move from a specific case to a valid generalization (Fritsche, 1997). Determining specifically what causes the rapid increase in default risk poses a great challenge.

As noted above, we assumed triangular distributions, which may or may not reflect distributions for a real company. In practice, distributions should fit real data (Potluri & Rajan, 2010). As a more general matter, the ranges must be reasonably good estimates. If our input distributions do not reflect reality our outputs will be invalid (Kolb, 2004, p. 324). However, single point estimates must also be good guesses. The objection about making good projections applies exponentially to the static model as there we are asked to project the exact point of each input.

We noted above the risk that with the static model one could manipulate the inputs to get the desired result. The same is true with our model, except that instead of one decision point being skewed an entire range of results would be skewed. We hope, however, that decision makers would make principled input choices.

The default trigger may be too harsh as it operates in an all or nothing fashion. In reality, investors inject more cash even in a bad year if they expect improvements in the near future. In addition, our model assumes the same result in a year where cash flow falls just below the cutoff as in a year where cash flow falls off disastrously. In the real world, there is likely more nuance in deciding whether to allow the firm to default, and if so, that would mitigate the debt cliff effect.
Determining an appropriate default trigger in practice may be difficult, especially for large companies. For small companies, the default trigger may be fairly easy to determine. A small entrepreneur may not have the resources to fund a cash shortfall and therefore the debt trigger may be close to zero. Most banks will not lend to a small company to pay mandatory long-term debt obligations. Troubled large firms have more options for raising cash, making the default trigger more difficult to determine.

It is difficult to overstate the breadth of applications of this kind of model. We have shown the application of MCS to finance and managerial accounting techniques. Entrepreneurs can better analyze new companies. A company can better adjust its long-term financial strategy to match its business strategy and operating environment. The traditional approach ignores variability, whereas Monte Carlo methods intuitively show a range of outcomes allowing for more informed decision-making and more efficient results.

Firms can also use MCS to better manage their projects and investments. Certain risks, like the danger of a default over time, do not readily appear in traditional models. For example, an airline or car company facing a cyclical environment could employ the model to determine the likelihood that high debt combined with a bad spell will at some point send them to bankruptcy court. Such a firm could then mitigate the risk by borrowing less, perhaps avoiding ruin for all its stakeholders--investors, creditors, employees, business partners, governments and communities. Lenders could better price their loans, reducing the risk of more publicly funded bank bailouts.

Investors could use the debt cliff phenomena as a bankruptcy detector. An investor could try to determine a default point then use historical data to develop distributions of revenue and expenses, allowing a forecast of the likelihood of default within a particular time frame. The model might reveal companies with unfair valuations allowing the investor to profit. Lastly, if the debt cliff actually exists,
awareness of the debt cliff could help firms, lenders and investors better avoid the costs associated with bankruptcy. Presumably, decision makers aware of risks would make better decisions than if they were ignorant of risks. If nothing else, they would at least understand the risks they are taking. Further research could reveal whether the debt cliff in fact describes the real world.

**Works Cited**


